

# Practical Grammar-based Procedural Modeling of Architecture

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SIGGRAPH Asia 2015 Course Notes

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# Abstract

This course provides a comprehensive, in-depth introduction to procedural modeling of architecture using grammar-based approaches. It first presents all necessary fundamentals and discusses the various advanced features of grammar languages in detail. Subsequently, context sensitivity, which is crucial for many practical tasks, and the different forms of support for it are addressed extensively. The course concludes by looking into several further advanced aspects, such as local edits or GPU-based variants.

Elements from a large body of work are covered and presented in a coherent, structured way. The course explores the range of solution approaches, provides examples, and identifies limitations; it also highlights and investigates practical problem cases.

The course is useful for practitioners and researchers from many different domains, ranging from urban planning, geographic information systems (GIS) and virtual maps to movies and computer games, with interests ranging from content creation to grammar-based procedural approaches in general. They learn about the arsenal of available techniques and obtain an overview of the field, including more recent developments. The audience benefits from a coherent treatment of ideas, concepts, and techniques scattered across many (sometimes lesser-known) publications and systems. This course helps in developing a realistic understanding of what can be done with current solutions, how difficult and practical that is, and with which tasks existing approaches cannot cope.





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# 1 Introduction

Course: Practical Grammar-based Procedural Modeling of Architecture

## Introduction

Michael Schwarz



## Procedural modeling

Model objects by specifying a procedure of how to construct/generate them

Different approaches/kind of procedures for different objects

*This course:*

### Grammar-based approaches

- Grammar = set of rules + ...
- Principle: successive refinement guided by these rules

### Shapes

- Primarily man-made structures encountered in architecture

## Example: rule-based modeling of facades

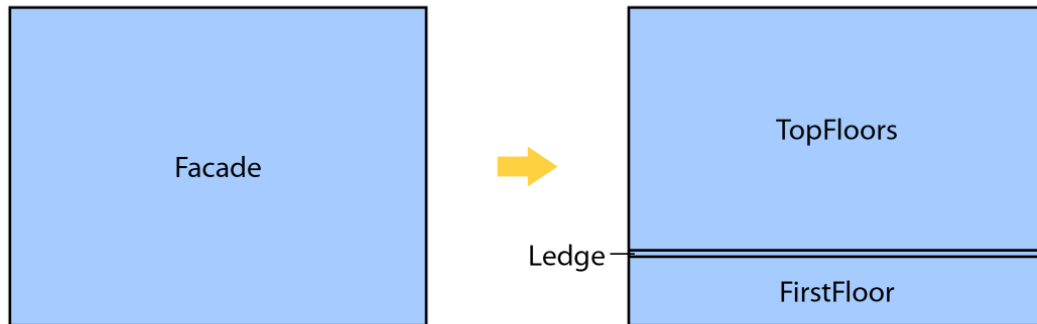


Real-world facade



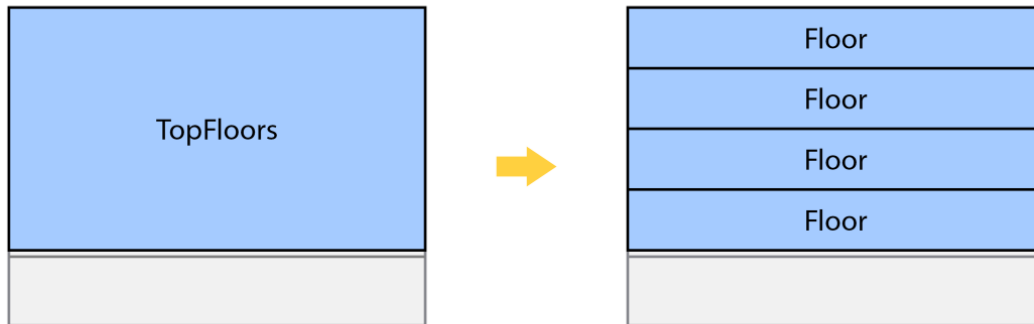
Vertical structure

## Example: rule-based modeling of facades



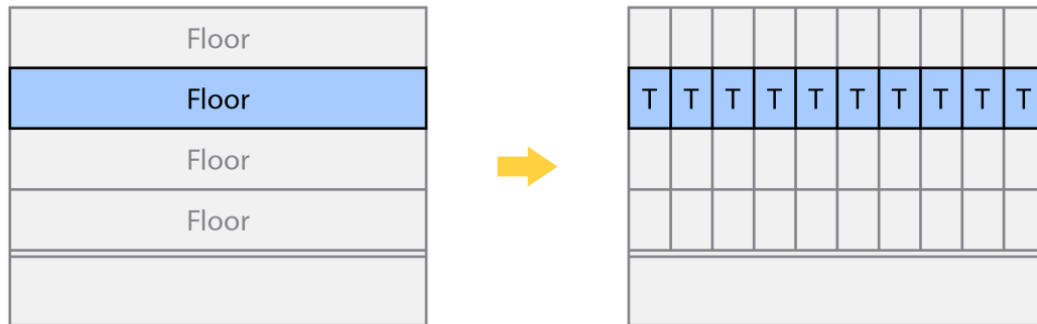
Rule: **Facade**  $\rightarrow$  split(y) { 3.5 : **FirstFloor** | 0.3 : **Ledge** | ~1 : **TopFloors** }

## Example: rule-based modeling of facades



Rule: `TopFloors`  $\rightarrow$  `repeat(y) { 3 : Floor }`

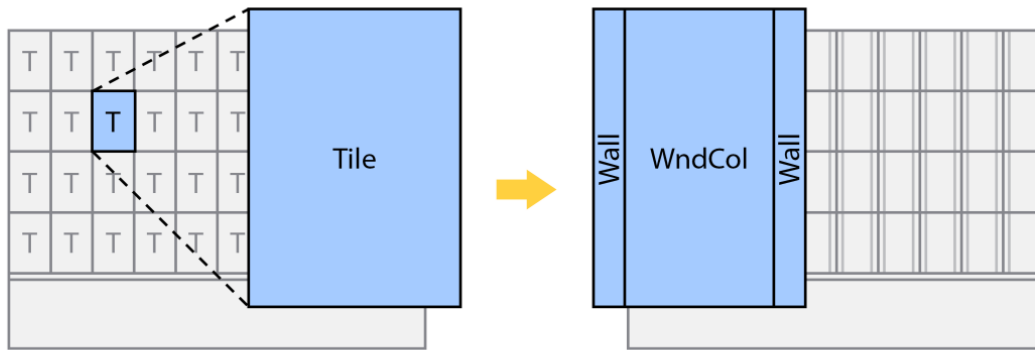
## Example: rule-based modeling of facades



Rule: **Floor**  $\rightarrow$  repeat(x) { 2 : **Tile** }

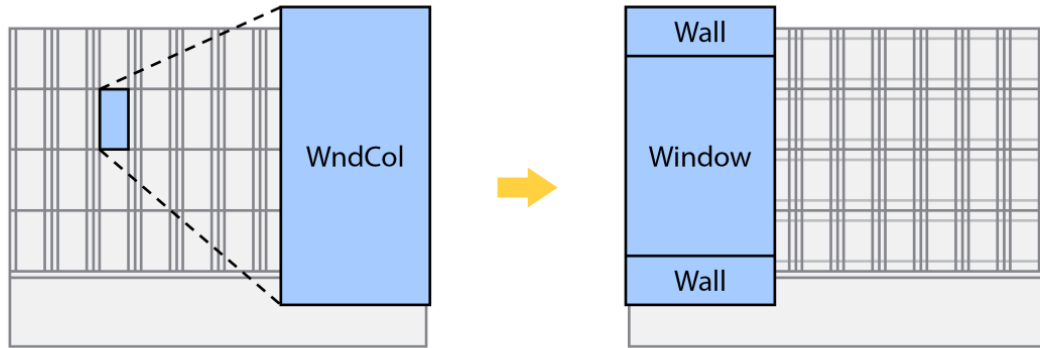


## Example: rule-based modeling of facades



Rule: **Tile**  $\rightarrow$  `split(x) { ~1 : Wall | 1.5 : WndCol | ~1 : Wall }`

## Example: rule-based modeling of facades



Rule:  $\text{WndCol} \rightarrow \text{split}(y) \{ \sim 1 : \text{Wall} \mid 2 : \text{Window} \mid \sim 1 : \text{Wall} \}$

## Properties and promises

<b>Scalability</b>	Large-scale generation of similar but varied objects <ul style="list-style-type: none"><li>• One potential answer to the ever-increasing demand for content</li></ul>
<b>Compactness</b>	Compressed representation <ul style="list-style-type: none"><li>• Example: building footprint + attributes + grammar</li></ul>
<b>Descriptiveness</b>	Describes the essence of a design ("recipe") <ul style="list-style-type: none"><li>• Can facilitate understanding and exploration</li></ul>
<b>Flexibility</b>	Adapt to different geometries and settings <ul style="list-style-type: none"><li>• Requires careful design</li></ul>
<b>Reusability</b>	"Model once, use many times"

## Applications

### **Movies & games**

*large-scale city scenes, ...*

### **Mapping**

*3D buildings from attributed footprints, ...*

### **Urban planning**

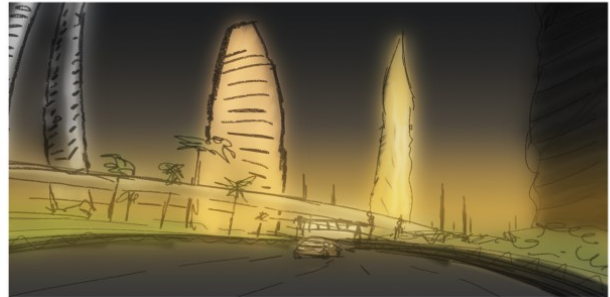
*visualization, analysis, exploration  
of different development strategies, ...*

### **Architecture**

*parametric building design, ...*

### **Archeology & cultural heritage**

*reconstruction, ...*



The images were kindly provided by Matthias Buehler ([matthias.buehler@mac.com](mailto:matthias.buehler@mac.com)).

## Applications

### Movies & games

*large-scale city scenes, ...*

### Mapping

*3D buildings from attributed footprints, ...*

### Urban planning

*visualization, analysis, exploration  
of different development strategies, ...*

### Architecture

*parametric building design, ...*

### Archeology & cultural heritage

*reconstruction, ...*



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## Applications

### Movies & games

*large-scale city scenes, ...*

### Mapping

*3D buildings from attributed footprints, ...*

### Urban planning

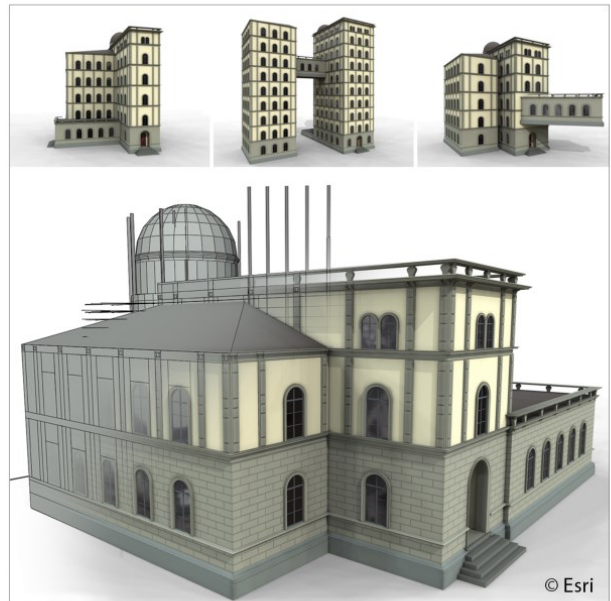
*visualization, analysis, exploration  
of different development strategies, ...*

### Architecture

*parametric building design, ...*

### Archeology & cultural heritage

*reconstruction, ...*



## Applications

### Movies & games

*large-scale city scenes, ...*

### Mapping

*3D buildings from attributed footprints, ...*

### Urban planning

*visualization, analysis, exploration  
of different development strategies, ...*

### Architecture

*parametric building design, ...*

### Archeology & cultural heritage

*reconstruction, ...*





## Example: Favela

(Matthias Buehler & Cyrill Oberhaensli)



A nice example that demonstrates what can be done with grammar-based procedural modeling techniques is the Favela project by Matthias Buehler ([matthias.buehler@mac.com](mailto:matthias.buehler@mac.com)) and Cyrill Oberhaensli. Among others, it deals with hilly terrain and sloped building footprints, includes procedural vegetation, features cables and clotheslines, and involves the distribution of connection points and detail assets.



## Example: Favela

(Matthias Buehler & Cyrill Oberhaensli)



© Matthias Buehler & Cyrill Oberhaensli

## Example: Favela

(Matthias Buehler & Cyrill Oberhaensli)



© Matthias Buehler & Cyrill Oberhaensli

## Scope and goals

Focus on **grammar-based** procedural modeling

- Not covered: related topics such as procedural road networks or content pipelines

Overview of available solutions and the state of the art

- Coherent treatment of various ideas, concepts, and techniques

Become familiar with

- Involved aspects
- Capabilities
- Limitations
- Interrelation between features
- Mode of operation
- Practical problem cases

Develop a realistic understanding: *What can be done?*

*How difficult and practical is it?*

## Procedural modeling systems

<b>CGA shape</b>	Müller, Wonka, Haegler, Ulmer, van Gool (2006)
<b>CityEngine (CE)</b>	Procedural/Esri
<b>Generalized grammar (G<sup>2</sup>)</b>	Krecklau, Pavic, Kobbelt (2010)
<b>CGA++</b>	Schwarz, Müller (2015)

### Many other systems and extensions with important contributions

- Often based on/influenced by CGA shape
- Unfortunately, details often omitted (e.g., syntax, semantics, derivation process)
- Examples: Lipp08, Thaller13, Schwarz14, Steinberger14

## 2 Fundamentals

Course: Practical Grammar-based Procedural Modeling of Architecture

### Fundamentals

Peter Wonka



## Formal languages

- A string over a set  $\Sigma$  (called **alphabet**) is a finite sequence of elements from  $\Sigma$
- We use lower case letters  $a, b, c, d, \dots$  to describe elements of the alphabet

## String grammars

- **Definition:** a grammar is a quadruple  $(NT, \Sigma, P, S)$
- NT – a set of non-terminal symbols
  - We use upper case letters A, B, C, ...
- $\Sigma$  – alphabet, a set of terminal symbols
- P – a set of productions rules
- S – start symbol

Example production rules:

S  $\rightarrow$  aSBC  
S  $\rightarrow$   $\epsilon$   
CB  $\rightarrow$  BC  
aB  $\rightarrow$  ab  
bB  $\rightarrow$  bb  
bC  $\rightarrow$  bc  
cC  $\rightarrow$  cc

## String grammars – Chomsky hierarchy

- Regular grammars
- Context-free grammars
- Context-sensitive grammars
- Unrestricted grammars



more general

- Context-free rules are the basis for most work in computer graphics and computer vision
- In computer graphics, these rules will be extended to add context to "context-free" grammars
  - In string grammars there is only 1d context; we need more general spatial context



## Context-free grammars

- Rules have the form

$$NT \rightarrow (NT \cup \Sigma)^*$$

- Example

$S \rightarrow \epsilon$

$S \rightarrow A$

$A \rightarrow aAdB$

$A \rightarrow abc$

$B \rightarrow b$

- Counter-example

$S \rightarrow aSBC$

$S \rightarrow \epsilon$

$CB \rightarrow BC$

$aB \rightarrow ab$

$bB \rightarrow bb$

$bC \rightarrow bc$

$cC \rightarrow cc$

## How to give grammars a spatial interpretation?

- L-systems
  - derive complete strings, interpret the string geometrically using turtle graphics
- Set grammars, CGA shape
  - interleave derivation and geometric interpretation

## L-systems

- Similar to string grammars
- Parallel derivation
- Successfully used for plant modelling
- [The Algorithmic Beauty of Plants \(1990\)](#)



Images: The Algorithmic Beauty of Plants 1990

## L-system example

- $\Sigma = \{F, +, -, [, ]\}$
- $F$  (starting symbol)
- $F \rightarrow FF-[-F+F+F]+[+F-F-F]$

### Geometric interpretation

- $F$ : go forward
- $+$ ,  $-$ : turn by  $22.5^\circ$
- $[$ ,  $]$ : push and pop the turtle on stack
- 4 iterations of replacement

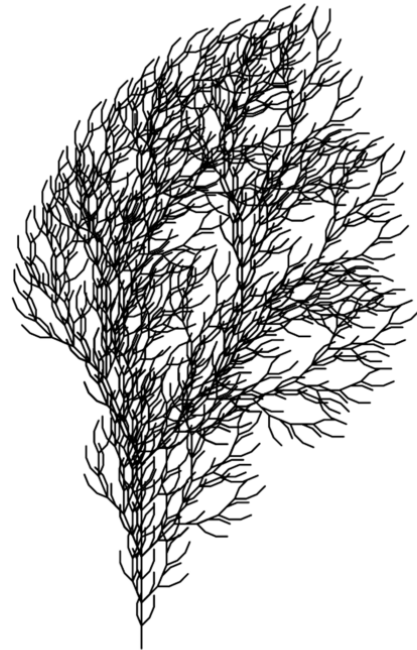


Image: The Algorithmic Beauty of Plants 1990

## Shapes

- Shape (CGA shape)
  - **Symbol** (for better readability we use labels instead of letters)
  - **Scope** (oriented bounding box)
  - **Geometry** (mesh, color, texture, shader attributes, ... )
  - **Parameters** (string, bool, double)
- Shapes can be **terminal** and **non-terminal**

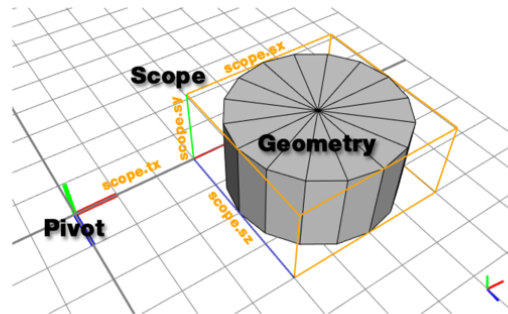


Image: CityEngine Online Help System

## Shape design choices

- Pivot (reference coordinate system) as shape attribute (CityEngine)
  - Extension for geo-spatial coordinate systems
- Shape types:
  - Only **boxes** in non-terminal nodes
  - **solid**, **boundary**, **empty** (Nil)

## Rules

- Rule Form:  
*PredecessorShape --> Successor*
- *PredecessorShape*: exactly one shape
- *Successor*: a sequence of **actions** generating zero to multiple shapes
- Actions can be
  - **shape operations**
  - **symbols**
- We also use the terms Left-Hand-Side (**LHS**) and Right-Hand-Side (**RHS**) / **rule body** of a rule

## Example rule

```
Lot --> s('0.8, '1, '0.8) center(xz) extrude(20) Envelope  
Envelope --> ...
```

- Non-terminal symbols: **Lot**, **Envelope**
- Shape operations: **s**, **center**, **extrude**

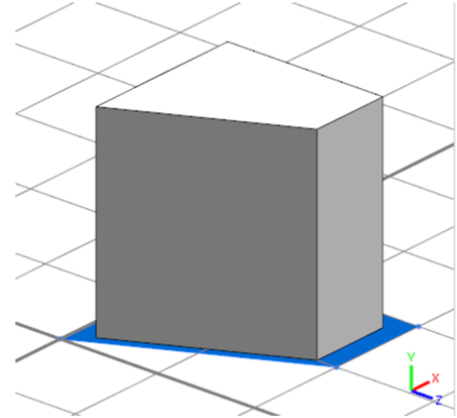


Image: CityEngine Online Help System



## Rule syntax examples

- CGA shape

```
floor --> Subdiv("X", 2, 1r, 1r, 2) { B | A | A | B }
```

- CityEngine

```
floor --> split(x) { 2: B | ~1: A | ~1: A | 2: B }
```

- G<sup>2</sup>

- Rules and parameters have types

```
$Facade : Box(th: Float, mh: Float, ...) ->  
    splitY( [bh, 0, :$BottomFloor[BF], ... , ... ] );
```

## Elementary shape operations

- Insertion of assets
  - Transformations
  - Extrusion
  - Center
  - Component split
  - Subdivision split
- 
- Most examples use CityEngine syntax

## Insertion/replacement

- Insertion operation:
  - `i("FILENAME")`
  - Bounding box of the mesh is scaled to the size of the scope per default
- Example:  
`Head--> i("beethoven.obj")`
- Details
  - Built-in shapes, e.g.  
`i("builtin:cube")`
  - Some grammars use insertion to transition from non-terminal to terminal symbol

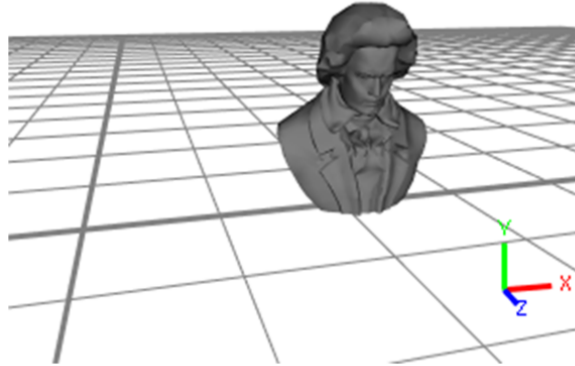


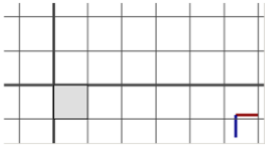
Image: CityEngine Online Help System

## Transformations

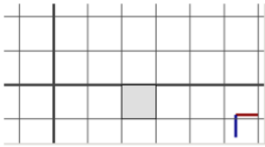
- Adapted from L-systems
- Translation: `t(tx,ty,tz)`
- Rotation: `r(rx,ry,rz)`
- Scale: `s(sx,sy,sz)`
- Advanced choices
  - Choice of scope, world, pivot, or object coordinate system
  - Augmenting current transformation vs. setting transformation
- Absolute
  - in model coordinates
  - e.g. `s(3, 3, 2)`
- Relative
  - proportional to the scope size
  - e.g.  
`s('0.5, '1, '1)`  
`s(0.5*scope.sx, 1*scope.sy, 1*scope.sz)`

## Transformation example

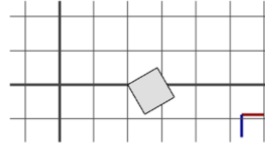
- A--> i("builtin:cube")



- A--> i("builtin:cube")  
t(2,0,0)



- A--> i("builtin:cube")  
t(2,0,0) r(0,30,0)



- A--> i("builtin:cube")  
t(2,0,0) r(0,30,0) t('2,0,0)

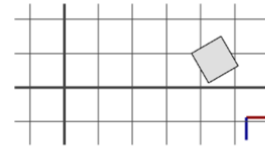
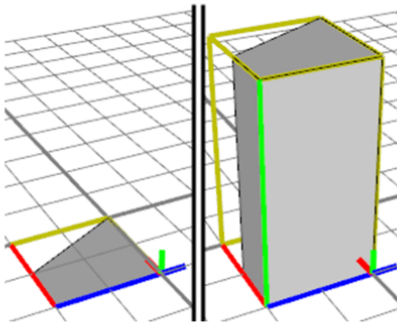


Image: CityEngine Online Help System

## Extrusion

- Extrude a flat shape

Lot --> `extrude(4)` Building



- Extruding along an axis

- e.g. lot is on a hill, not aligned with the ground plane

Lot --> `extrude(world.y, 30)`

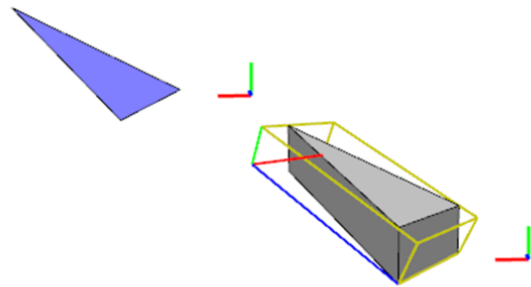


Image: CityEngine Online Help System

## Center

- Centering a scope:  
`center(axes-selector)`

- Example:  
Lot --> `s('0.8, '1, '0.8)`  
`center(xz) extrude(20)`

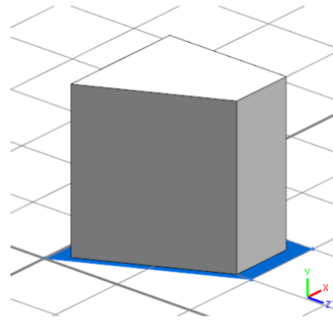


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## Component split

- CGA shape
- Splitting a mesh into its individual faces
- Component split has the form:  
`comp(comp-selector) { selector : actions | selector : actions ... }`
- Comp-selector:
  - faces (**f**), edges (**e**), vertices (**v**)
- Selectors:
  - `front, back, left, right, top, bottom`
  - `vertical, horizontal, aslant, nutant, side, all`



## Component split example

```
Building -->
  comp(f) {
    front : color("#ff0000") Main |
    side  : color("#0000ff") Side
  }
```

z-axis of new scope is normal to the face plane

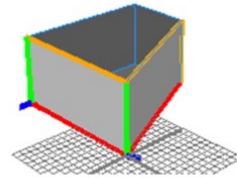
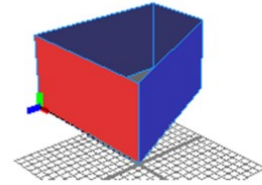
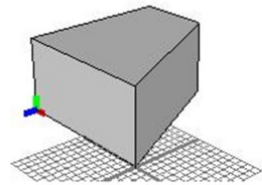


Image: CityEngine Online Help System

## Subdivision split

- Wonka 2003
- Control grammar to distribute parameters
- Grid split

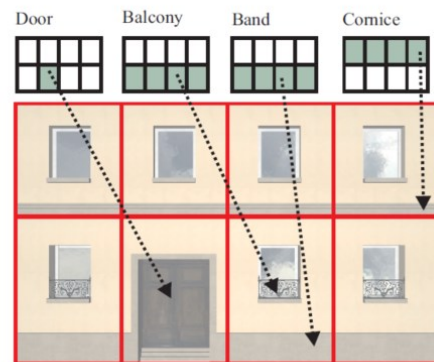


Image: CityEngine Online Help System

## Subdivision split

- CGA shape:
  - simplification to 1d splits, no control grammar
- Rule Format

```
split(axis) { selector : actions }
```
- Example:

```
Lot --> s(5,1,1) i("builtin:cube")
  color("#84c0fc") split_example02
split_example02 --> split(x) {
  3 : X
  | ~1 : X(cuboid_height1)
  | ~1 : X(cuboid_height2)
  | ~1 : X(cuboid_height3)
}
```

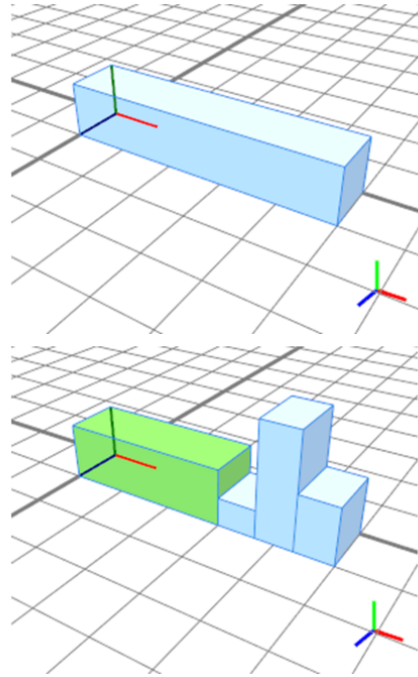


Image: CityEngine Online Help System

## Lack of space

- First, the absolute values get priority from left to right, e.g.

```
split_example03 --> split(x)  
{ 2:X | 1:A | 1:Z | 2:Y | 1:Z }
```

- Second, relative values (~) divide remaining space according to their weights

```
split_example03 --> split(x)  
{ 1.5:X | ~3:Y | 1.5:X | 0.5:X }
```

- Not all specified shapes might be generated

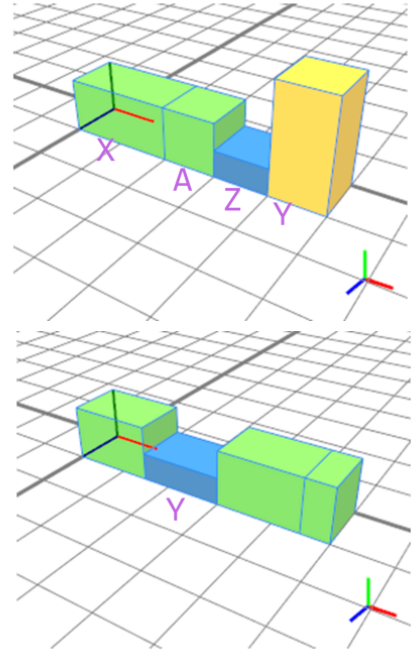


Image: CityEngine Online Help System

## Splitting design choices

- What splitting axis are allowed?
  - Axis-aligned splits
  - General 3D axis
  - General splits
- Is splitting of arbitrary geometry supported?
  - Splitting of 2D geometry (for lots)
  - Splitting of 3D geometry
  - Splitting of boxes only
- Grid Split vs. 1D split

## Repeat split

- Wonka 2003 / CityEngine example

- Denoted by \*

- Example:

```
ex01 --> split(x) {  
  1: X(3) |  
  { ~1:Y | 0.2:X | ~1:Y }* |  
  1: X(3)  
}  
(initial scope has size 10)
```

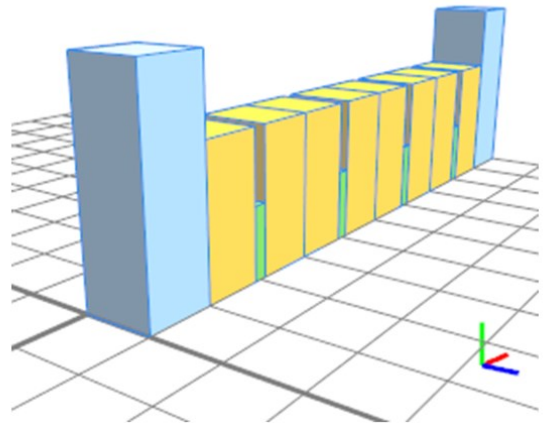


Image: CityEngine Online Help System

## Combining repeat splits

- How to combine repeat splits?
- On the same level
  - E.g. `ex01 --> split(x) { {1: A}* | { 1: B}* }`
  - How to express that `AABBBB` is preferred over `AAAABB`?
- Nested
  - E.g. `ex02 --> split(x) { 1: A | { 1: B}* }*`
  - How to express that I would like to have as many inner as outer repeats?
  - E.g. `AB`, `ABBABB`, `ABBBABBBABBB`

## Shape tree

- Shape tree: tree of shapes generated by the derivation process

Lot -->

```
s('0.8, '1, '0.8) center(xz) extrude(20) Envelope
```

```
Envelope --> split(y) { ~4 : Floor. }*
```

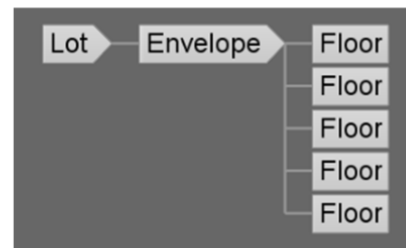
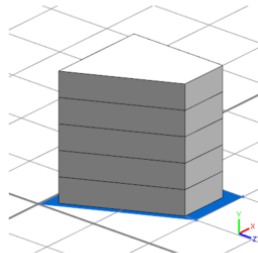
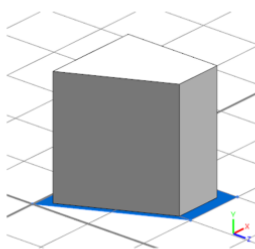


Image: CityEngine Online Help System



## Graph-based representations

- **Shape operations** are represented as nodes
- Nodes have inputs and outputs
- Data flow is controlled by edges
- Examples: Silva et al., Thaller et al., Patow, Houdini

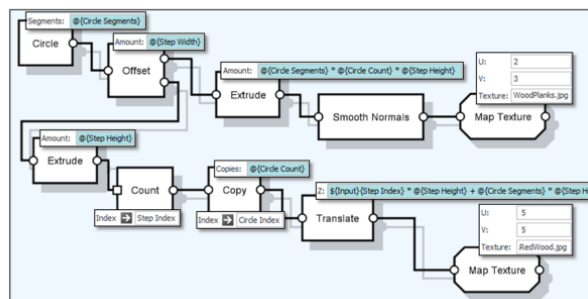


Image: Silva 2015

## Parametric rules

- Rules can have a list of parameters

- E.g.

```
Lot --> s('0.8','1','0.8) center(xz) Footprint(20)  
Footprint(height) --> extrude(height*0.8) Envelope
```

## Conditional rules

- Conditional rules have the form

```
PredecessorShape -->  
  case condition1:  
    Successor1  
  case condition2:  
    Successor2  
  
  ...  
  else:  
    SuccessorN
```

## Conditional rule example

```
Footprint(type) -->  
  case type == "residential" || type == "park":  
    case geometry.area/2 < 200 && geometry.area > 10:  
      extrude(10) Envelope  
    else:  
      extrude(15) Envelope  
  case type == "industrial":  
    extrude(100) Factory  
  else:  
    NIL
```

## Stochastic rules

- Stochastic rules have the form

```
PredecessorShape -->  
  percentage%: Successor1  
  percentage%: Successor2  
  ...  
  else: SuccessorN
```

- Example:

```
Lot -->  
  30%: Lot("residential")  
  20%: Lot("retail")  
  else: Lot("industrial")
```

## Recursion

- We call a grammar **recursive** if a shape in the derivation tree can have a shape with the same label / symbol as ancestor

- Examples:

```
Floor --> split(x) { 3: WinTile | ~1: Floor }
```

```
A --> BC
```

```
B --> DE
```

```
D --> AF
```

```
...
```

- Note: not all systems allow recursive grammars

## Derivation process

- Depth first, e.g.  $G^2$ 
  - always replace the first non-terminal
  - $S, AB, DEB, dEB, deB, debb$
- Breath first (sequential)
  - $S, AB, DEB, DEb, dEb, db$
- Breath first (parallel / L-systems)
  - $S, AB, DEbb, dbb$
- **Problem:** Derivation strategy changes the outcome, if rules can query the global context

- Example Grammar

$S \rightarrow AB$   
 $A \rightarrow DE$   
 $B \rightarrow bb$  (if B is next to A)  
 $B \rightarrow b$  (otherwise)  
 $D \rightarrow d$   
 $E \rightarrow e$  (if E is next to d)  
 $E \rightarrow nil$  (otherwise)

## Guidance of derivation order

- Priorities (CGA shape)
  - each rule has a priority assigned
- Evaluation phases (Steinberger2014)
  - Sort the rules into multiple stages called [evaluation phases](#)
  - Queries are only allowed to ask about state of previous or the same evaluation phase
- Construction stages (Schwarz2014)
  - new operation [stage\(\*k\*\)](#)
  - shapes with smallest stage have priority
- Events (CGA++)
  - Coordinating the derivation with a complex event system
- Approximate breadth-first derivation using heuristics



## Strategies for parallel implementation

- Object-level parallelism:
  - A city has many objects, e.g. buildings, derive each building in parallel
- Shape-level parallelism:
  - derive different shapes of the same object (building) in parallel
  - e.g., after some initial derivation, derive mass models, floors, windows, ... in parallel
- Rule-level parallelism:
  - Parallelize different parts of a rule
  - E.g. `building --> [t(...) mass1] [t(...) mass2] [t(...) mass3]`
- Operation-level parallelism:
  - Parallelize different parts of the same operation
  - E.g. `floor --> split(x) { 1: A | ~1: B | ~1: B | 1: A }`



### 3 Features of grammar languages

Course: Practical Grammar-based Procedural Modeling of Architecture

## Features of Grammar Languages

Michael Schwarz



## Operations

Purpose: modify or subdivide the current shape

*Previous part:*

### **Elementary operations**

- Scope modifications
- Split & repeat
- Component split

*This part:*

### **Advanced/complex operations**

- Geometry creation
- Roofs
- Further subdivisions
- Geometry manipulation

## Geometry creation

## Operations

### Create new shape geometry

- Usually based on current shape geometry

### Examples

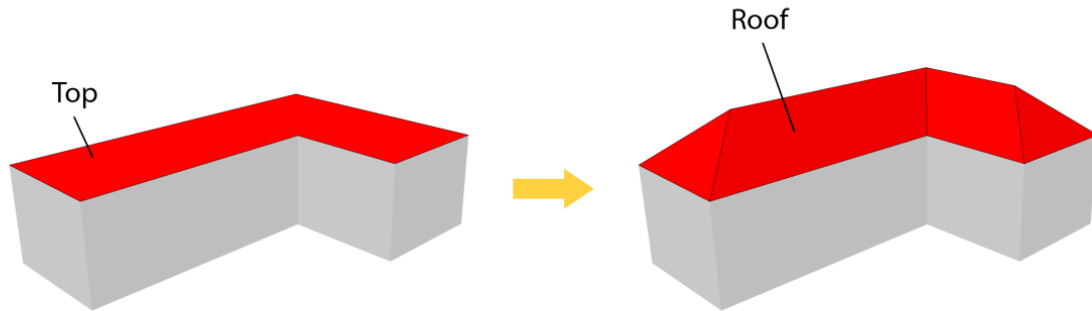
- Create pre-defined shape (e.g., circle)
- Load geometry from asset
- Explicit constructors
- Extrusion: `extrude(amount)`
- Find inscribed rectangles (e.g., `innerRect (CE)`)
- Erect roofs

## Roofs

## Operations

Dedicated operations for selected roof types

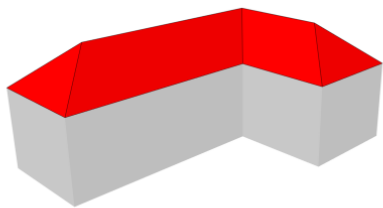
- Erect a roof on each face of the current shape
- ➔ Current shape is turned into a roof



Top → roof(...) Roof

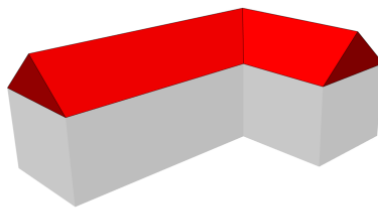
## Roof types

## Operations



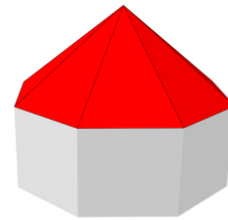
cross-hipped

CE: `roofHip(...)`



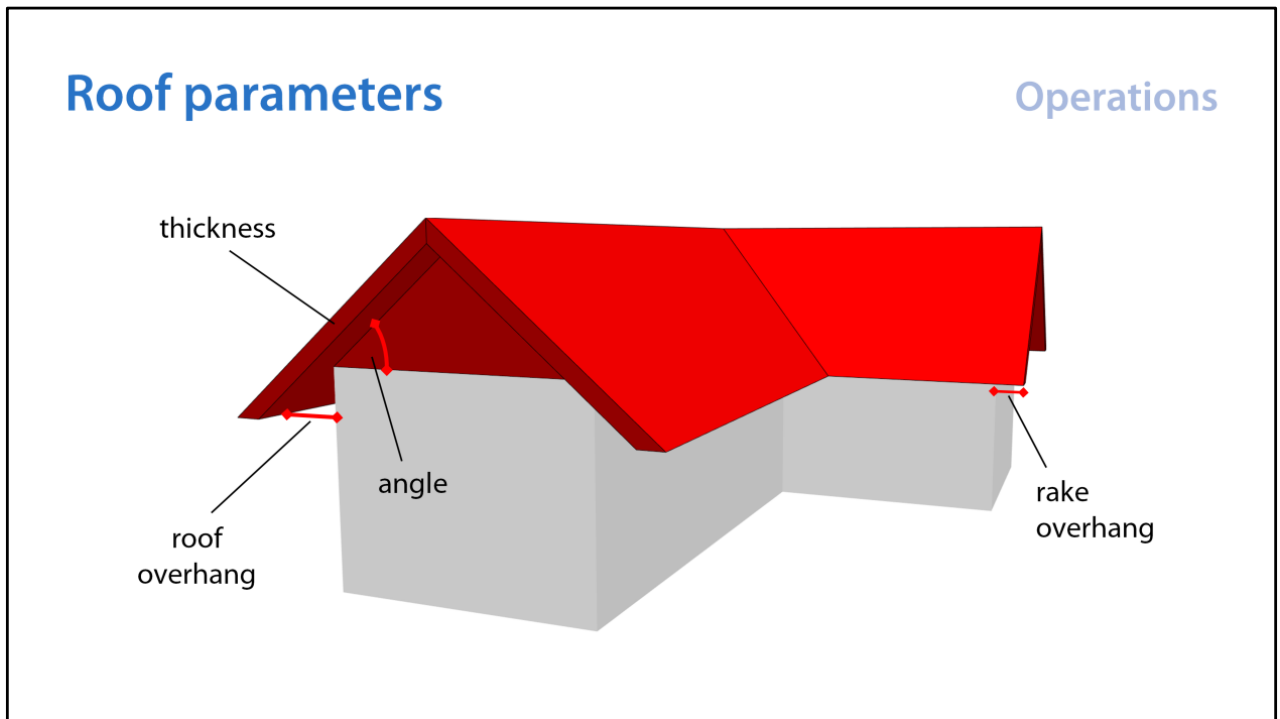
cross-gabled

`roofGable(...)`



pyramid

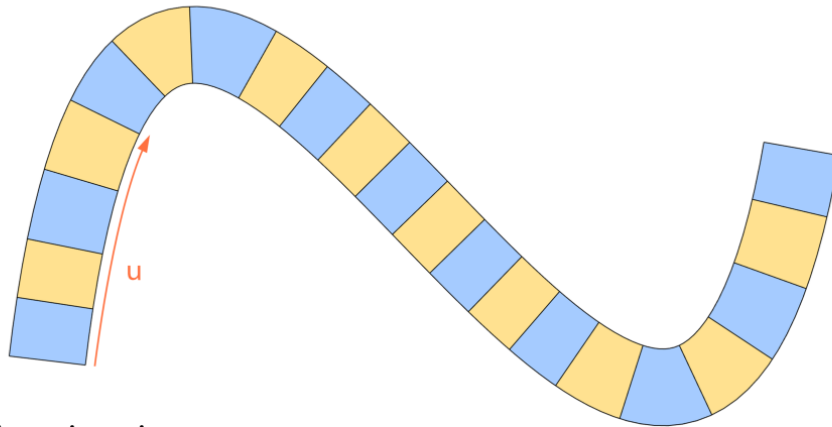
`roofPyramid(...)`



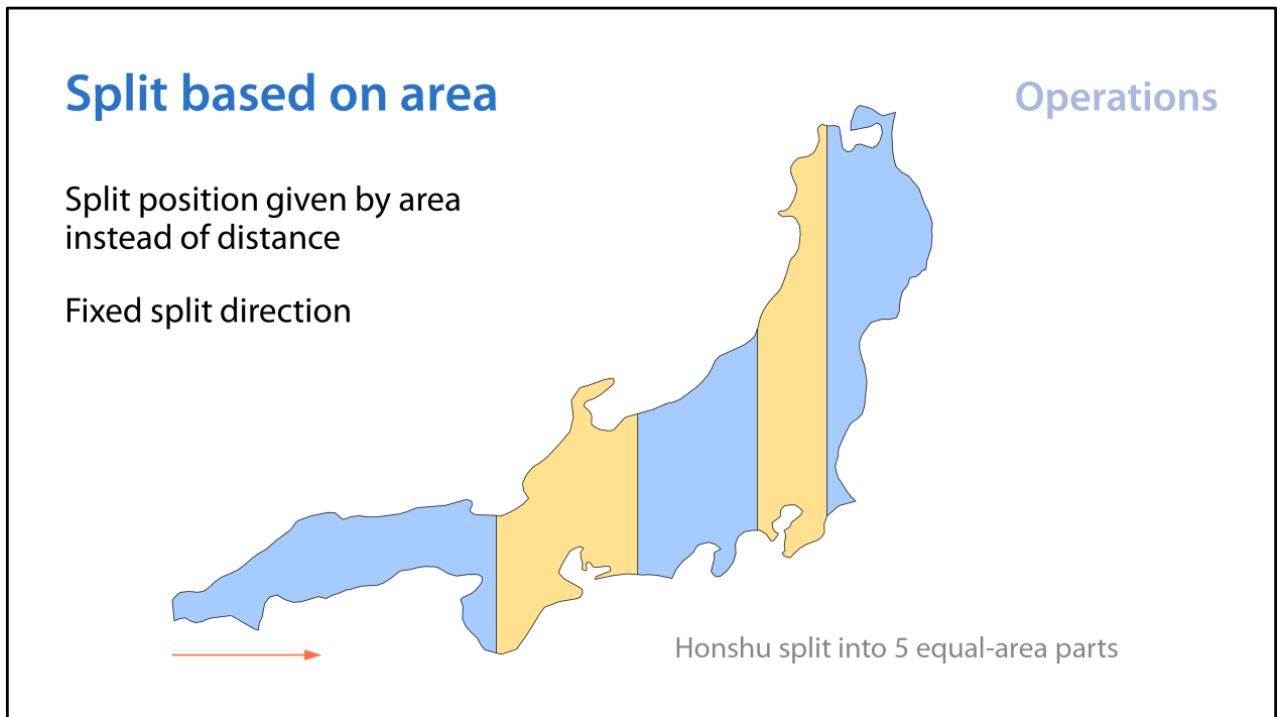


## Split based on texture coordinates

Operations



Split position given in texture space



## Offsetting

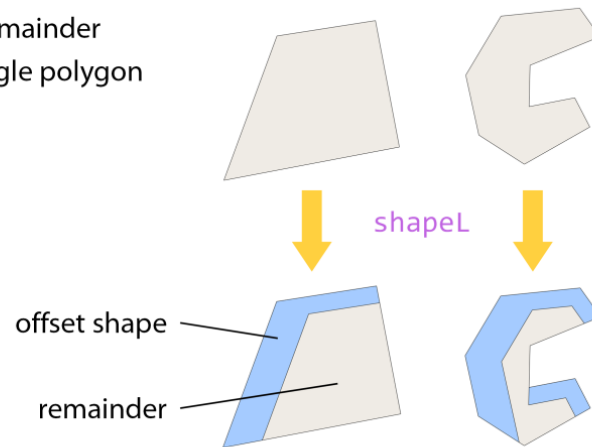
Offset selected edges by a certain distance

- Result: one offset polygon per edge + remainder
- Offset polygons may be merged to a single polygon

### Examples

- `offset`, `setback` (CE)
- `shapeL`, `shapeU`, `shapeO` (CE)
- `frame-split` (Thaller13)

## Operations



frame-split: uniform inward offset, partitioning similar to straight-skeleton-based approach

## Geometry manipulation

## Operations

Manipulate/transform current shape geometry

### Examples

- Reverse normals
- Remove collinear vertices
- Remove holes
- Split non-convex faces into convex parts
- Simplify geometry for lower level of detail
- Compute or transform texture coordinates

## Managing code complexity

@ expression level

### Constants

- Defined at global scope
- May encode input/design parameters
- May be exposed in UI (CE: "attributes", attr)

```
const floorHeight = 2.4
```

### Functions

- Essentially a named expression
- May have parameters
- Defined at global scope
- May use dynamic scope during evaluation (CE)

```
scopeVolume = scope.sx  
              * scope.sy  
              * scope.sz
```

(CE)

## Managing code complexity

@ rule level

### Modules ( $G^2$ )

- Rule with sub-rules
- Parameters of rule are accessible by sub-rule
- Sub-rules live in new namespace
- Rule prefixes

:	Sub-rule
—	Global rule
../	Sub-rule of parent module

```
$Rule:Box(w:Float, h:Float)
-> repeatX(w, :$SubRule1);
{
  $SubRule1:Box
  -> repeatY(h, $SubRule2);
  $SubRule2:Box
  -> ...
}
```

## Managing code complexity

@ file level

### Sub-grammars (CE)

- Content from other grammar can be imported

```
import id : filename
```

- Imported rules, functions & constants become visible with prefix *id*.

```
id.SomeRule(id.someFunc(...), id.someAttr)
```

- Values of "attribute" constants in imported grammar may be overwritten

*e.g., with value of "attribute" constant in importing grammar of same name*

## Ease of expression

### Local variables (CGA++)

- Can increase readability
- Help reusing expression values (especially random choices)

```
R -->
  with(a = rand(4, 9),
        b = someFunc(a))
{
  A(b) B(70 - b) C(a)
}
```

- **Emulation** possible:

Turn variables into parameters

```
R          --> R1(rand(4, 9))
R1(a)      --> R2(a, someFunc(a))
R2(a, b) --> A(b) B(70 - b) C(a)
```

**But:** avoiding unwanted side effects  
on shape tree can be challenging



## Ease of expression

### Conditions

- Possibilities often limited:

	CGA shape	$G^2$	CE	CGA++
Anywhere within rule?	✗	✗	✗	✓
Nesting possible?	✗	✗	✓	✓
Combinable with stochastic selection?	First condition, then stochastic		✗	✓

- Working around the limitations can be tedious; often involves introducing additional rules/functions and duplicating code

## Values/objects within grammars

### Elementary types

- Numbers (floats)
- Booleans
- Strings

### “Producers”

- Functions (CGA++)
- Rules ( $G^2$ , CGA++)

### Collections

- Lists (CGA++)  
*elements of same type, variable size*
- Tuples (CGA++)  
*elements of different types, fixed size*
- “Containers” = multi-dimensional lists ( $G^2$ )

### “Product”

- Shapes (CGA++)

CE has functions for representing a list as a string, where elements are separated by a semi-colon.

## Rules as values

(G<sup>2</sup>, CGA++)

A rule may be used just as every other value; e.g., passed as argument

### Using named rules (CGA++ syntax)

- Rule:  
`Building(h) --> ...`
- Reference:  
`%Building`
- With fixed argument(s):  
`%Building(20)`

### Rules as parameters: G<sup>2</sup> specifics

- Non-terminal symbol  
*parameter of type rule*
- Abstract structure template  
*module/rule with rule(s) as parameter(s)*
- Use as value: prefix with `@`

At least in CGA++, rules are full first-class citizens and hence cannot only be passed around but may also be stored in collections or as shape attributes.

## Rules as values

### Anonymous rules (CGA++)

- Can be defined in-place  
`%< t(5, 0, 0) ... >`
- May have parameters  
`%(h)< extrude(h) ... >`
- May be empty  
`%<>`
- Rule value captures values of all outside variables referenced within body  

```
map(h:list(7, 11),
    %< extrude(h) >)
≡ list(%< extrude( 7) >,
      %< extrude(11) >)
```

## Rules as values

### Operations (CGA++)

- Invoke rule

`invoke(%Building, 5)`  $\equiv$  `Building(5)`

- Execute rule in-place

`apply(%(h)< extrude(h) >, 10)`  $\equiv$  `extrude(10)`

- Stop rule execution

`A B stop C D`  $\equiv$  `A B`

## Shapes as objects

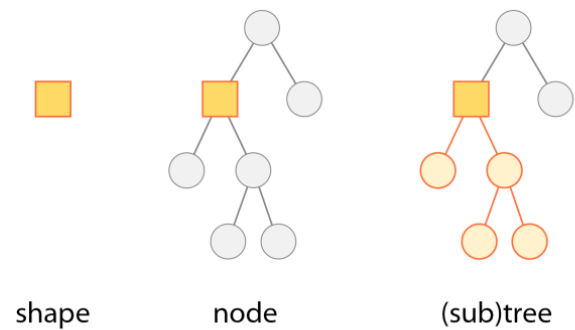
(CGA++)

### First-class citizenship

- Existing shapes can be used as values
- New shape values can be created

### One entity, three views

- Shape itself
- Corresponding node in shape tree
- (Sub)tree rooted in that node



## Accessing existing shapes

(CGA++)

### Current shape

- `this`

### Scoped labels

- Definition:  
`label = action`
- Access:  
`label` (within same rule body)  
`parentShape::label`

### Shape tree queries

- Simple navigation  
`parent(node),`  
`children(node),`  
...
- More complex queries  
`findAll(tree, predicate, traversal),`  
...

expression evaluated for each node

e.g., `"bfs"` = breath-first

## Shapes as arguments

(CGA++)

### Operations

- Boolean operations
  - `intersect(otherShape),`
  - `minus(otherShape),`
  - `union(otherShape),`
  - ...
- ...

### Functions

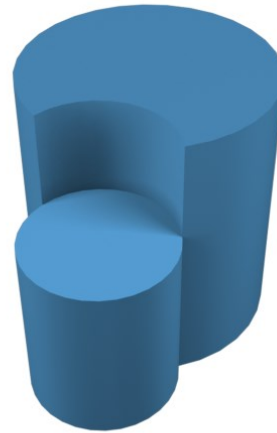
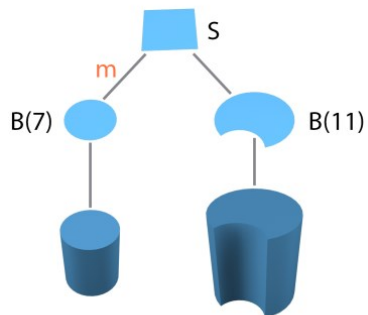
- Geometric properties
  - `area(shape),`
  - ...
- Spatial relationships
  - `overlaps(shape1, shape2),`
  - ...
- ...



## Illustrative example

(CGA++)

```
S    --> i("circle") m=B(7) t(3, 0, 0)
        s(10, 0, 10) minus(m) B(11)
B(h) --> extrude(h)
```



## Creating new shapes

(CGA++)

### Functions

- Take shape value(s) as input, return new shape value(s)
- Shape modification  
`t(tree, dx, dy, dz), ...` *translates all shapes in subtree*
- Subdivision  
`split(shape, axis) pattern, ...` *returns list of part shapes*
- Tree rewriting  
`refine(tree, rule), ...` *applies rule to all leaf nodes*  
|  
*expression evaluated for each leaf node*

## Creating new shapes

(CGA++)

### Tree constructor

- Syntax: `< actions > (base)`
- Initiates a sub-derivation process  
with start shape *base* and  
start rule `%< actions >`
- Yields a shape tree

### Operations for incorporating shapes

- Embed a shape tree as sibling of  
the current shape  
`include(tree)`
- Modify current shape to match  
another one  
`adopt(shape)`

## Use case: temporary/auxiliary shapes

(CGA++)

### Construct shapes on-the-fly

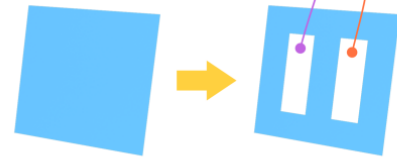
- to reason about them *e.g., to derive parameter values*
- to use them as arguments *e.g., for multi-shape operations*

### Example

```
S --> minus(list(this->t('0.2, 0, '0.2)->s('0.2, '1, '0.6),
                  this->t('0.6, 0, '0.2)->s('0.2, '1, '0.6)));
```

syntactic sugar: *chain operator*

```
this->t('0.2, 0, '0.2)
≡ t(this, '0.2, 0, '0.2)
```



## Use case: exploring different alternatives

(CGA++)

### Example

- Two different development schemes

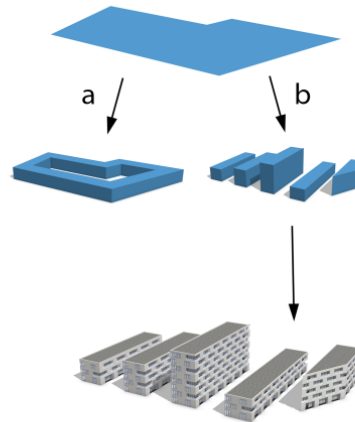
```
a = < DesignA >,  
b = < DesignB >
```

- Choose the one which results in larger total mass volume

```
V(a) > V(b)
```

- Refine shape tree of chosen option and embed it

```
include(refine(...))
```



Example is adapted from Figure 4 of the CGA++ paper.

## Enhanced operations

(CGA++)

Selectors can become arbitrary predicate expressions

- Predefined selectors are exposed as (local) functions on shapes (or local variables)
- Implicit variables provide information about objects tested
- Function values are applied implicitly

### Example: component split

```
comp("f") {
  top($shape)      : Roof
| right || front   = FrontCorner
| side && $index > 4 : Wall1
| !bottom          : Wall2
}
```

*implicit variable*

*or simply: top*  
 $\equiv$  right(\$shape) || front(\$shape)

## Beyond “normal” shapes

Generalization of shapes beyond scope + mesh geometry:

### Non-terminal classes ( $G^2$ )

- Each shape (non-terminal) belongs to a class \$A:Box -> ...
- A class defines operations and attributes
- Box, FFD, FFD Turtle, Mesh, Polygon, Triangle, ...

#### Box

- Scope part of a traditional shape
- Attributes: transformation + size
- Operations for creating terminals  
renderGeometry(filename)

#### FFD

- Trilinear freeform deformation cage
- Strings of FFDs can approximate curves
- May be created by operations of Box  
cornerFFD(angle, \$SomeFFDRule)

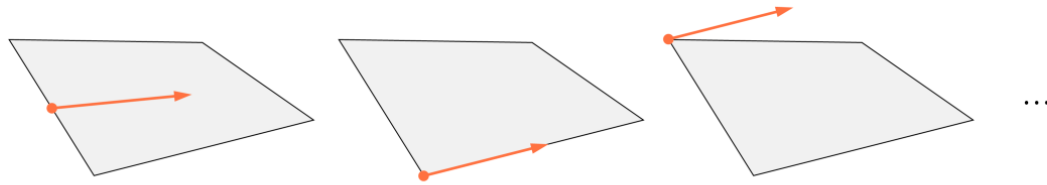
## Beyond “normal” shapes

Generalization of scopes beyond bounding boxes:

**Convex polyhedral scopes** (Thaller13)

- Scope can approximate geometry more faithfully
- May benefit simplicity of expression
- Affects semantics/degrees of freedom of operations

*Example: direction and start position for splitting*





## 4 Context-sensitive modeling

Course: Practical Grammar-based Procedural Modeling of Architecture

### Context-sensitive Modeling

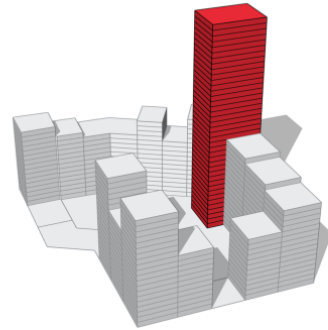
Michael Schwarz



## Tasks involving context sensitivity

### Selection/Identification

- Identify largest footprint  
*sizes may only be known after decomposition of parcel*
- Identify highest building mass  
*heights may have been chosen stochastically*
- Select exactly  $k$  random footprints  
*number of footprints may not have been known a priori*

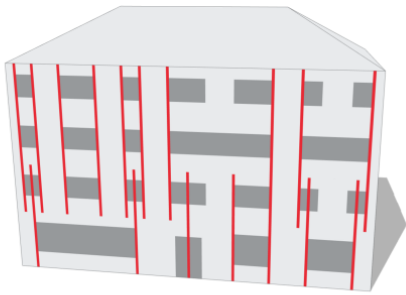


### Analysis

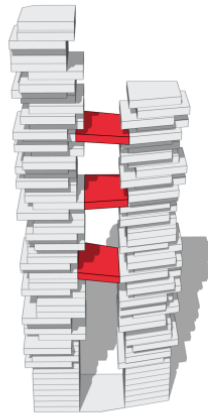
- Determine number of footprints
- Determine total area

## Tasks involving context sensitivity

### Alignment



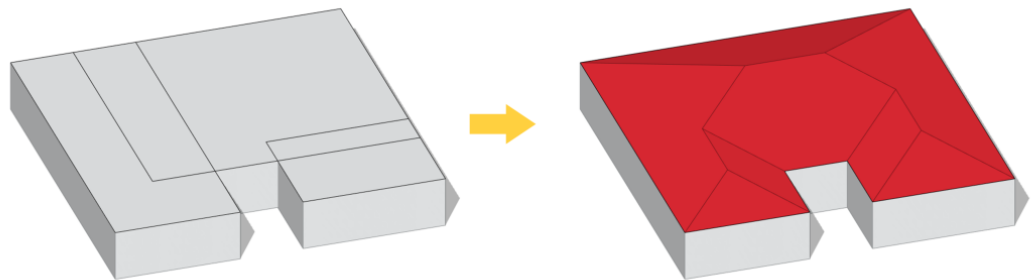
### Interconnections



## Tasks involving context sensitivity

### Boolean operations

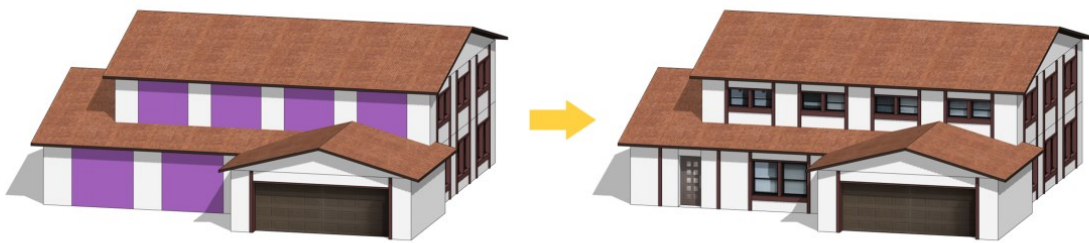
- Cut out intersection with overlapping shapes
- Merge overlapping building masses
- Create single top surface spanning multiple masses and erect coherent roof



## Tasks involving context sensitivity

### Account for occlusion

- Place door in unoccluded tile
- Adjust windows vertically to fully fit into unoccluded space



## Attributes

Encode specific information about a shape

- Can carry semantic and context information
- Value accessible within grammar

### Built-in attributes

- Example: position of scope's origin  
`scope.tx`, `scope.ty`, `scope.tz` (CE)
- Some may only be queried but cannot be (directly) set

### User-defined attributes

- Example: floor index

## User-defined attributes

### Boolean: **Flags** ( $G^2$ )

- Can be set when creating a successor shape  
`$Successor[MyFlag]`
- Cannot be cleared, remain set in whole sub-tree
- Can be queried  
`Flag.MyFlag`
- Evaluate to false unless explicitly set

## User-defined attributes

### Globally defined attributes (CE)

- Defined at global scope  
`attr floorHeight = 2.4`
- Accessible as variable  
`floorHeight`
- Value can be overwritten by an operation  
`set(floorHeight, 3.0)`
- Overwritten value applies to current shape and all its successors
- Built-in attributes (basically) exposed identically



## User-defined attributes

### Key-value pairs (CGA++)

- Attribute name (key) can be an arbitrary string
- Value can be of any type (including shape)
- Highly flexible — *e.g., allows arbitrary number of attributes*
- Set attribute: `set("floorHeight", 3.0)`
- Retrieve value: `get("floorHeight", 2.4)` fallback value

### Detail: child shapes in shape tree don't inherit attributes

- Retrieval walks up the ancestor chain until the attribute is found
- Advantage: can identify shapes that had an attribute explicitly set  
*e.g. allows tagging specific nodes*

## Context information from operations

### Encoding options

- Built-in attribute  
*set for each successor shape*  
`split.index` (CE)
- Special variable  
*visible to operation's arguments*  
`Operator.index` ( $G^2$ )

### Examples

- Split operation (CE)
  - `split.index`, *index of successors*
  - `split.total` *number of successors*
- Component split (CE)
  - `comp.sel`, *selector*
  - `comp.index`, `comp.total`
- Extrusion ( $G^2$ )
  - `Operator`
  - `.index` *index of edge*
  - `.la` *edge's left outer angle*
  - `.ra` *edge's right outer angle*

## Involvement of other shapes

Context-sensitive modeling often requires referring to other shapes

**Prerequisite:** these shapes must exist when establishing the context

Important role: **derivation process/order** and available **means to guide it**

They influence

- what contexts are possible
- whether the set of involved shapes is well defined and deterministic/reproducible
- the actual effort required to ensure that shapes exist

## Example implications of derivation approaches

### Purely sequential, depth-first execution ( $G^2$ )

- Particularly limited influence on derivation order

### Evaluation phases (Steinberger14)

- Coarse-grained, global synchronization points at rule level
- Support referring to shapes from earlier phases  
(and, in simple settings, from the same phase)

### Events (CGA++)

- Flexible synchronization points (variable scopes, fine-grained) at action level
- In principle, any derivation order could be enforced
- Enable the modeler to locally express ordering dependencies

## Identifying involved shapes

### Different mechanisms and strategies

- Offer different granularity and control
- Some only yield a (conservative) set of candidate shapes — *e.g., for occlusion testing*
- Others identify specific shape(s) — *e.g., for alignment*

### Examples of simpler/coarser options

- Select by symbol name  
*all shapes with that symbol*
- Select by relationship in shape tree  
*e.g., ancestors, siblings, or siblings of ancestor*
- Select by construction stage  
*all shapes available at a certain stage; wait until stage reached*

## Identifying involved shapes

(CGA++)

**First-class support for shapes** enables arbitrary selections

- Directly query the shape tree
- Take a shape resulting from a preceding action in the same rule
- Explicitly pass a specific shape to a rule as argument or store it as an attribute

Powerful option: identification by **participation in events**

- An event is raised with the operation **event**
- An event serves as **synchronization point**, thus influencing the derivation order to ensure existence
- The scope of an event may be restricted to a subtree via **event groups**
- All shapes participating in an event instance are available to the **event handler** as a list of shape values

## Collecting shapes during derivation

### Approach (Krecklau11)

- Shapes are stored in a container (= multi-dimensional list)
- Container is passed as argument to rules
- Container's content can be modified within a rule (call-by-reference semantic)
  - modify existing entry*
  - add new entry via `container.push(value)`*
- Entries and their order are well defined due to  $G^2$ 's sequential, depth-first execution
- **Use case:** once collection is completed, create interconnections between shapes

**Note:** compiling such a collection also possible with CGA++

- Different derivation process requires different approach
- Collection only **after** the shapes to collect have been derived

## Dedicated support for selected tasks

Often, systems are per se not expressive enough for dealing with most context-sensitive tasks

Common solution: offer ad-hoc functionality for a few selected tasks

### Examples

- Occlusion
- Snapping
- Trimming



## Occlusion queries

### CGA shape

CGA shape offers **query function** `Shape.occ(occluderSet)`

- Result: "none", "part", or "full"
- Test may be used in a rule's condition

Potential **occluder sets**:

- "all" *all shapes generated so far*
- "active" *all active shapes, i.e., all leaves of the current shape tree*
- "noparent" *all shapes except current shape's ancestors*
- "symbolName" *all shapes with that symbol*

## Occlusion queries

CGA shape

Visibility computation may be controlled by additional arguments

- Example: `Shape.occ(occluderSet, "distance", enlargementAmount)`  
*enlarges the current shape for the occlusion test*

Variant: **sightlines**

- Test for occlusion of shortest line to certain geometry
- Example: `Shape.visible("street")`

**Limitation:** concrete occluder set depends on actual derivation order

- Rule priorities may not offer enough control
- Sets often only deterministic in the case of known sequential derivation semantics
- Offering differently defined sets could be one remedy

## Occlusion queries

CityEngine

CityEngine offers multiple **query functions**

- `overlaps()`
- `touches()`
- `inside()`

Set of shapes considered as **occluders**

- Only shapes with a closed surface ("volumes")
- Only leaf shapes and (non-ancestor) shapes subjected to a component split
- Shapes from the (previous) final shape tree, not of the current, evolving shape tree
- Not the current shape and its successor shapes

## Occlusion queries

CityEngine

Actually two occluder pools

- **Intra-occluders:** occluders from “same” shape tree
- **Inter-occluders:** occluders from shape trees of other initial shapes
- Occlusion queries may be restricted to one of them

**Process:** up to 3 derivation passes

1. **Determine inter-occluders:** run derivation process for all initial shapes ➔ “ghost models”  
*no occluders considered by occlusion query functions*
2. **Determine intra-occluders:** run derivation process again ➔ “ghost shape tree”  
*only inter-occluders considered by occlusion query functions*
3. **Create final model:** run derivation process again (start with pruned ghost shape tree)  
*both inter-occluders and intra-occluders considered by occlusion query functions*

## Occlusion queries

CityEngine

### Issues

- Involves repeating the derivation process up to three times
- Inter-occlusion ignores intra-occlusion-induced effects in other shape trees
- Non-first-level intra-occlusion allowed but may not be resolved “correctly”  
*occlusion is evaluated with respect to the ghost shape tree,  
i.e., differences induced by the actual first-level intra-occlusion test results are ignored*
- Only very limited, coarse selection of occluders possible

## Occlusion queries

### Limitations of ad-hoc solutions

- Restricted to certain specific occluder sets
- Available occluder sets often too coarse-grained and/or hard to control

### Alternative to special support: make the grammar language more powerful

#### Example: CGA++

- Arbitrary options to identify occluders to test against
- Test with spatial query functions
- Allows deterministic and correct results
- Allows avoiding unnecessary shape derivations
- But: more grammar writing effort for cases covered by languages with dedicated support

## Occlusion

Occlusion queries only tell the degree of occlusion

- Actual occluders remain unknown
- Limits possible reactions

Unless the language is powerful enough:  
enabling a certain more advanced reaction requires  
an according special operation

**Example:** remove all occluded parts of the current shape

- New split operation `unoccludedParts` (Schwarz14)
- Combines occlusion test with Boolean operation

## Snapping

(CGA Shape)

**Goal:** coherent alignment of elements

**Approach:** adjust split positions  
such that they align to close-by lines/planes

### Realization

- Emit snap shapes via operation `Snap(axes, label)`
- Enhance split operations `Subdiv` and `Repeat` to account for these snap shapes
- Snapping behavior is enabled with special axes "`XS`", "`YS`", and "`ZS`"
- Considered snap shapes may be limited to those with a certain label

`Repeat("X", ...)`  
*without snapping*



`Repeat("XS", ...)`  
*with snapping*





## Snapping

Related approach: **avoidance volumes** (Thaller13)

- Adjust split positions such that overlap with certain shapes is avoided
- Allowed movement of split position is bounded by maximum distance
- Enhanced split operations take a list of shapes to avoid
- **Application:** avoid placing interior walls behind windows

## Trimming

(CE)

Component split into faces yields **trim planes**

- One for each shared edge, bisecting the dihedral angle between the faces
- Planes belong to/are a property of a shape
- Planes (shape-locally) encode information about the adjacent faces (at split time)

A shape's geometry can be trimmed by the shape's trim planes

- Operation **trim**
- Considers only enabled trim planes  
*controlled via built-in attributes **trim.horizontal** and **trim.vertical***

Trim planes are also considered by operation **i**

- Loaded geometry is cut by set of enabled trim planes

## Trimming

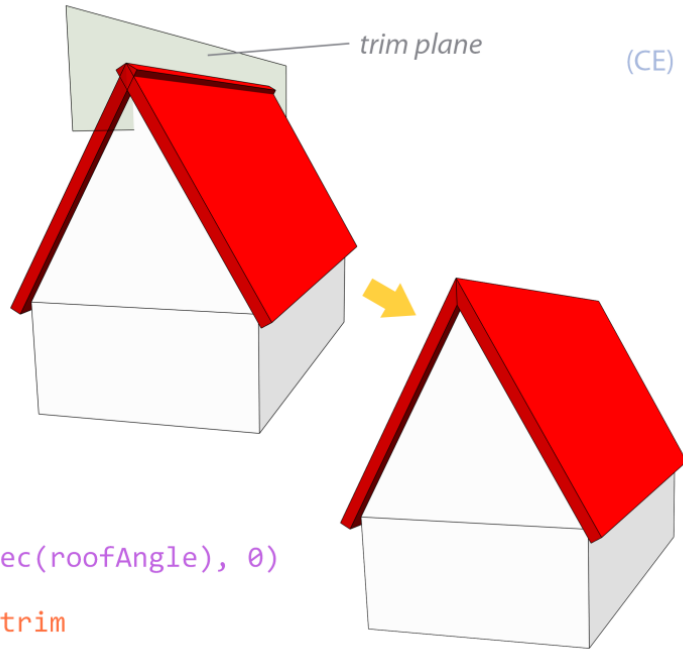
**Example:**  
Cutting extruded roof planes

*emits trim planes*

```

Roof -->
  comp(f) { ... : RoofPlane }
RoofPlane -->
  translate(rel, world, 0,
            roofThickness * sec(roofAngle), 0)
  extrude(-roofThickness)
  set(trim.horizontal, true) trim

```



(CE)

```

...
roofGable(roofAngle,
          overhangX + roofThickness * tan(roofAngle),
          overhangY)

```

Roof

...

```

square(x) = x*x
sec(x) = sqrt(1 + square(tan(x)))

```

## Spatial queries

Functions analyzing the spatial relationship of shapes provided as arguments

### Overlap and containment tests (CGA++)

- `overlaps(shape1, shape2)`
- `touches(shape1, shape2)`
- `inside(shape1, shape2)`

### Ray shooting (Krecklau11)

- `shoot(source, targets, yaw, pitch, alpha, beta, range)`
- Operates on rectangles
- Shoots a ray from the center of `source` in the direction given by `yaw` and `pitch`
- Returns the closest sub-rectangle of `targets` with the same size as `source`

## Operations involving multiple shapes

### Spectrum of use cases

#### Subtract other occluding shapes

- Just a local refinement of the shape
- Ordinary operation, taking other shapes as argument: `minus(otherShapes)` (CGA++)

#### Establish connection between two shapes

- Select one shape as initiator, establish connection from it to other shape
- Becomes part of the refinement of one shape again
- Ordinary operation, taking other shape as argument: `connectTo(targetShape)` (CGA++)

#### Merge multiple shapes via Boolean union

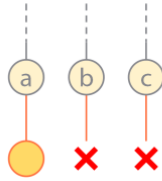
- Affects refinement of all involved shapes

## Merging multiple shapes

### Approach 1:

#### Coordinated refinement of multiple shapes

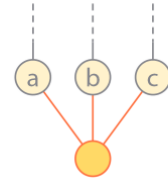
- Select one shape (a) as "master"
- Merge other shapes (b, c) into this shape  
*ordinary operation*
- Abandon the other shapes  
*replace by empty shape*
- Involves just ordinary operations



### Approach 2:

#### Replacement of multiple shapes by one shape

- Shape tree becomes a shape graph  
*How to deal with multiple parents?*  
*Which properties are inherited from whom?*
- Need new refinement mechanism  
*Multi-shape rules?*



## Merging multiple shapes

### Approach 1:

#### Coordinated refinement of multiple shapes

- Main requirement: other shapes must already exist when master shape is refined with multi-shape operation
- Realization benefits from language support for multi-shape coordination
- Simple solution possible with events  
*Identify shapes by participation in event*  
*Issue the respective update operations in the event's handler*

### Approach 2:

#### Replacement of multiple shapes by one shape

##### Idea: multi-shape rules

- Operate on a set of shapes  
*How to specify that set?*
- E.g., non-context-free rules (Thaller13)  
*Symbol\* ~ ...*  
*Selection of shapes by symbol limits possible use cases*
- Unclear when to apply rules

## Multi-shape operations

### Boolean operations (CGA++)

- `union(shape(s))`
- `intersect(shape(s))`
- `minus(shape(s))`

*create multiple connections  
at a time  
current shape plays no role  
motivated by underlying  
derivation process*

### Adding interconnections

- `connectTo(targetShape)` (CGA++)  
*create a connecting tube to the target shape;  
both shapes must be polygons*

*list of pairs of rectangles*

- `beam(correspondences, rule, stiffness, gravity, step, threshold)` (Krecklau11)  
*connect pairs of rectangles,  
creating a deformable beam for each*
- `chain(correspondences, segments)` (Krecklau11)  
*connect pairs of rectangles,  
creating a rigid chain for each*



## Multi-shape coordination

Most systems: no coordination across multiple shapes possible

- Refinement decision are performed locally for a shape
- Even if other existing shapes may be consulted:  
cannot (directly) influence their further refinement

Usual consequence:

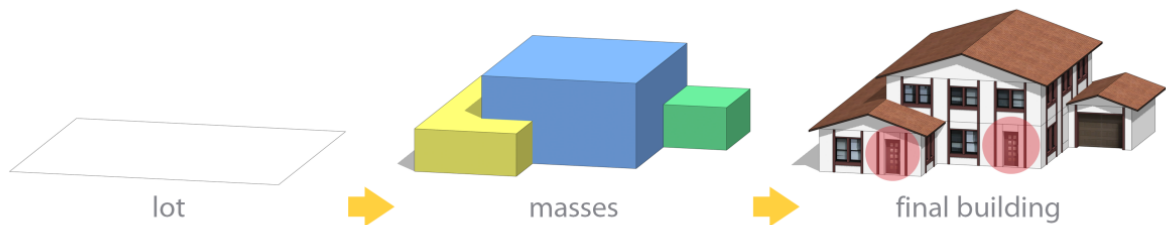
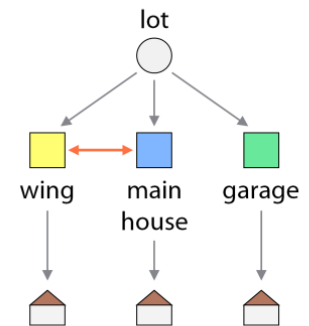
Any decision affecting multiple shapes has to be made  
no later than when refining their closest common ancestor

- Shapes themselves don't exist yet
- Must manually infer those properties of these shapes that influence the decision
- Easily becomes impractical, especially if stochastic elements are involved

## Need for multi-shape coordination

### Example

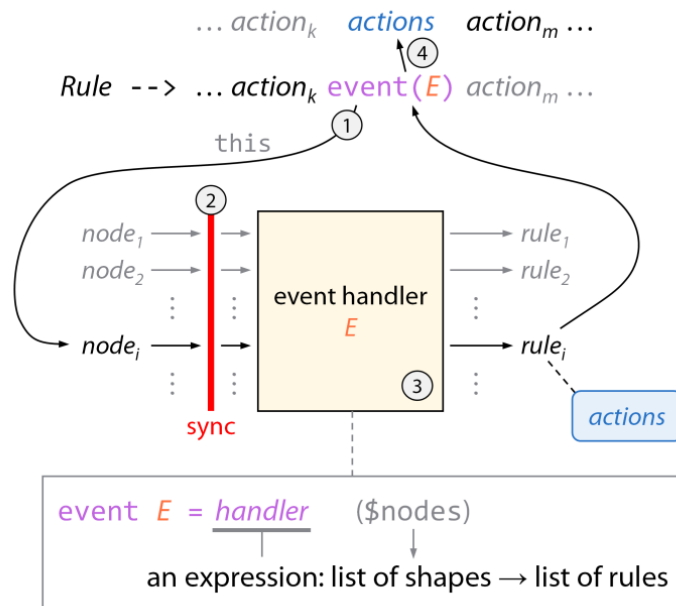
- Masses are placed randomly on lot
- Relative position and occlusion decide whether main house or wing should have door
- Making decision already when refining lot is not practical
- Would like to make decision once masses have been created



## Events (CGA++)

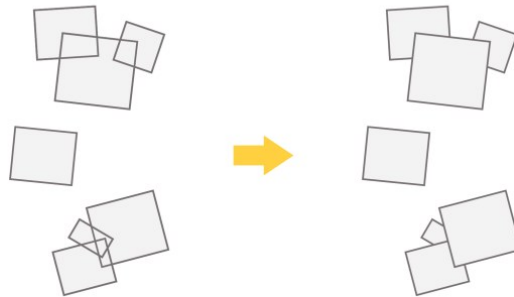
### Procedure

1. Operation **event** raises an event  
*suspends current derivation branch*
2. Wait until all derivation branches  
have raised some event
3. Consult event handler  
*input: the current shapes of all  
participating branches*  
*outputs a rule for each branch*
4. Resume derivation branch  
*first executes the returned  
rule in-place*



## Example: overlap resolution

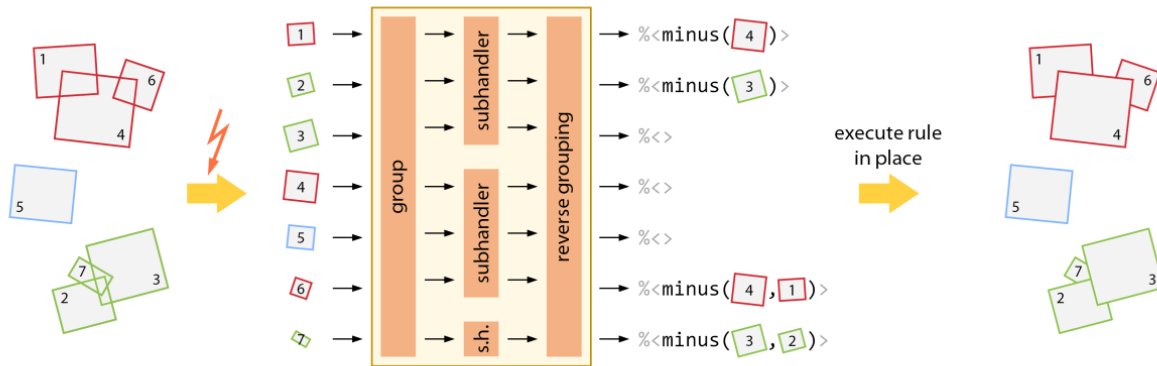
Events



For each shape: subtract overlapping larger shapes

## Example: overlap resolution

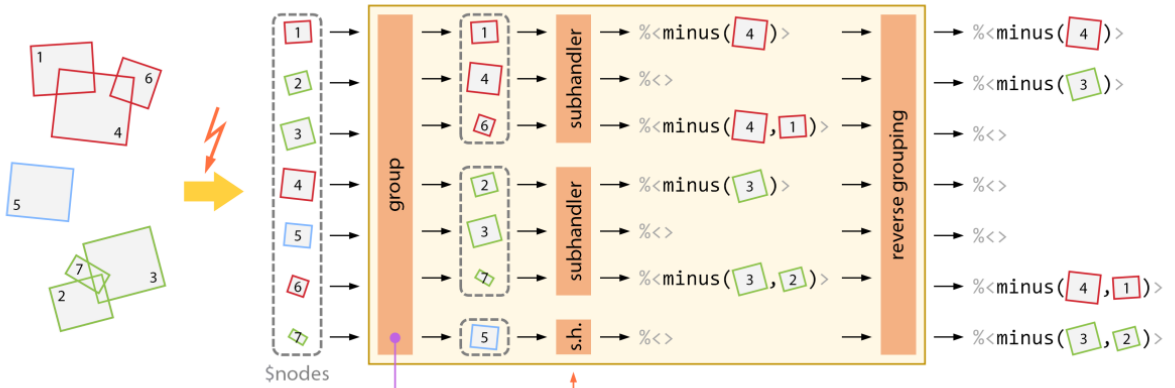
### Events



```
event ResolveOverlaps =
  partitionByPred($nodes, overlaps($a, $b),
    subtractLargerOnes($groupNodes))
```

## Example: overlap resolution

### Events



```
event ResolveOverlaps =
    partitionByPred($nodes, overlaps($a, $b),
                    subtractLargerOnes($groupNodes))
```

*predicate*

## Example: overlap resolution

Events

### Event handler

```
event ResolveOverlaps =  
  partitionByPred($nodes, overlaps($a, $b),  
    subtractLargerOnes($groupNodes))
```

### Subhandler

```
func subtractLargerOnes(shapes) =  
  with(byArea = sort(shapes, area($a) > area($b)),  
    isLargest = [s](index(byArea, s) == 0),  
    largerOnes = [s](sublist(byArea, 0, index(byArea, s))),  
    select(s : shapes) {  
      !isLargest(s): minus(largerOnes(s))  
    } )
```

## Events

(CGA++)

### Applications

- Coordinate further refinement
- Identify related shapes
- Influence derivation order
- Ensure existence of other shapes

*within event handler,  
output according actions*

*participation in the same event*

*place synchronization point,  
waits for other derivation branches*

*participation in the same event,  
place synchronization points*



## Events

(CGA++)

### Properties

- Enable [synchronization](#) among multiple derivation branches
- Enable [communication](#) among multiple shapes
- Enable making collective, [coordinated decisions](#) on how to proceed individually

### Note:

- Ability to synchronize among multiple derivation branches enables multi-shape coordination
- In principle, could repeatedly execute the same decision process (using all involved shapes), once for each affected shape
- Events with their handlers make this significantly simpler

## Event handler

(CGA++)

### Characteristics

- Arbitrary expression, yielding a list of rules (of same size as the input list of shapes)
- Facilitates reuse & compositing
- Enables dynamic grouping and hierarchical handling

### Convenience handler functions

- Offer a concise syntax for common use cases

```

• select(s : shapes) {
  | condition1 : actions1
  | condition2 = subhandler2
  | ...
}

```

*index of "master" shape*

```

• forall(shapes, "union", 0) {
  | postUnionActions
}
• foreach(s : shapes) { actions }

```

## Events: advanced features

(CGA++)

### Scoping mechanism: event groups

- Events operate globally by default, but can be made local to a subtree via event groups
- Operation `group(name)` creates a special group node; all shapes created by succeeding actions become descendants
- Specifying a group name when raising an event makes the event instance local to the subtree rooted in the closest matching group node ancestor
- Different instances of an event (e.g., one for each floor) can coexist

### Signaling

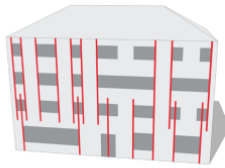
- Handling an event indicates that a certain stage has been reached
- Operation `wait` allows waiting until an event got handled for the first time (within a certain subtree)

## Task: alignment

*Simple option:*

**Use built-in snapping** (CGA shape)

- Define dominant lines and planes
- Perform snap splits
- Offers only limited control
- May not be powerful enough  
*e.g., no center alignment*



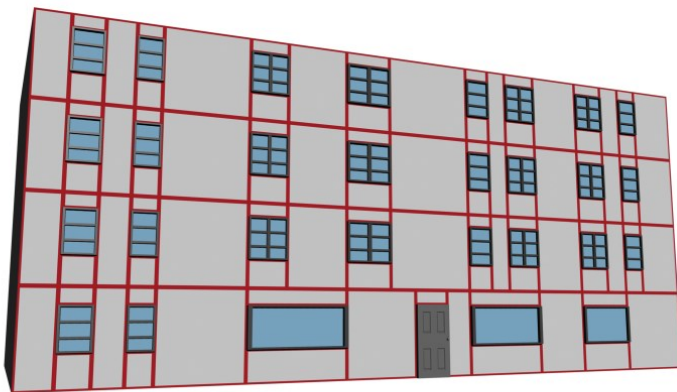
*Alternative:*

**Compute yourself**

- Query other shapes (if supported)
- Compute sizes, positions, split distances, etc.
- Requires “some” effort  
*but solution may be reusable*
- + Offers utmost flexibility

## Task: alignment

Non-trivial example (with CGA++)



- Center-aligned door
- Other elements aligned on their left and their right
- Respects also size and frequency constraints
- Elements randomly selected from candidate pool

## Task: cope with overlapping geometry

*Simple and limited:*

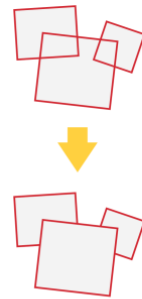
### Detect with occlusion query

- Supported by several systems
  - Provides only rather coarse information
  - Possible reactions rather limited
- e.g., do not place an element (such as a window) if occluded*

*Powerful:*

### Remove overlaps with Boolean operations

- Easy to implement with events
- Resulting shapes may have a form that is difficult to deal with



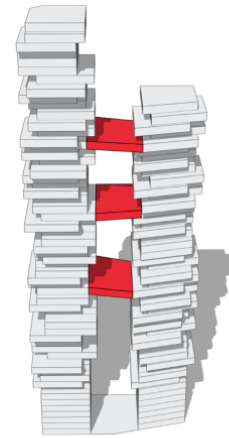
## Task: interconnections

### Step 1: determine connection partners

- Collect potential connection sites
- Determine correspondences
- $G^2$  (Krecklau11): collection of all candidates done during derivation
- *With events*: candidates could simply be the event's participants  
*but it is also possible to gather them from the current shape tree*

### Step 2: create connecting geometry

- $G^2$  (Krecklau11): given a list of correspondences, create all connections with a single operation
- CGA++/CE: during refinement of one connection partner, establish connection to other shape via operation (e.g., `connectTo`)  
*operation may have been emitted by event handler*



Examples: Favela project (external solution), Krecklau11, Schwarz15





## 5 Advanced aspects

Course: Practical Grammar-based Procedural Modeling of Architecture

### Advanced Aspects

Peter Wonka



## Visual editing of rules and parameters: challenges

- How to create a visual user interface to edit a single rule from scratch?
- How to create a visual user interface to edit the parameters of rules?
- How to create a visual user interface to edit the combination of rules?
- How to avoid chaos when naming the rules?

## Rule naming

- What is named a window?
  - Outer frame included?
  - Balcony included?
  - Blinds included?
- Naming rules consistently is difficult

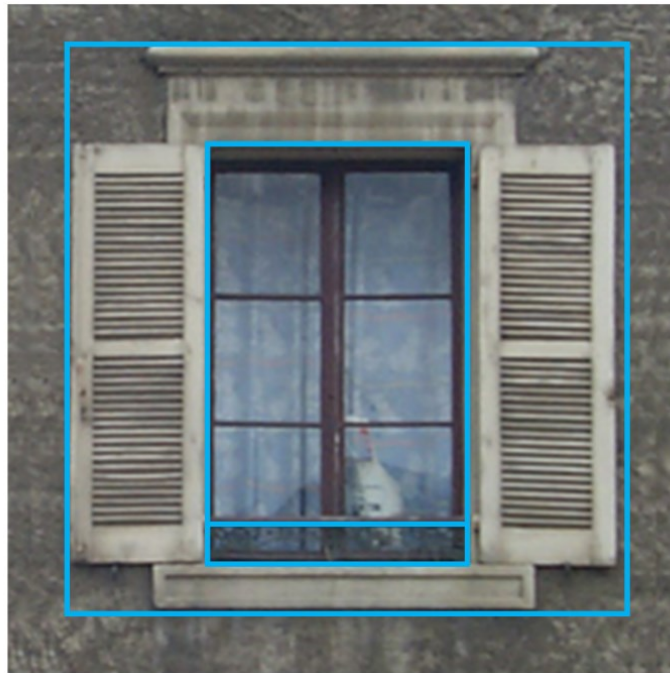


Image: Wonka

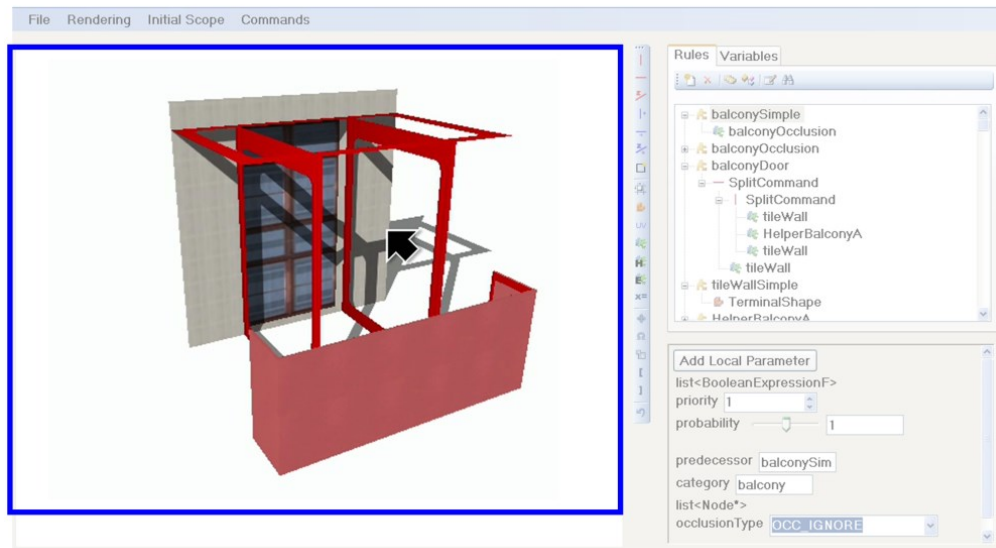
## Visual editing of rules and parameters: solutions

- Visual Rule Editor
- Graph-based rule editors
- Manipulators
- Procedural high-level primitives
- Styles
- Design Galleries

## Visual rule editing

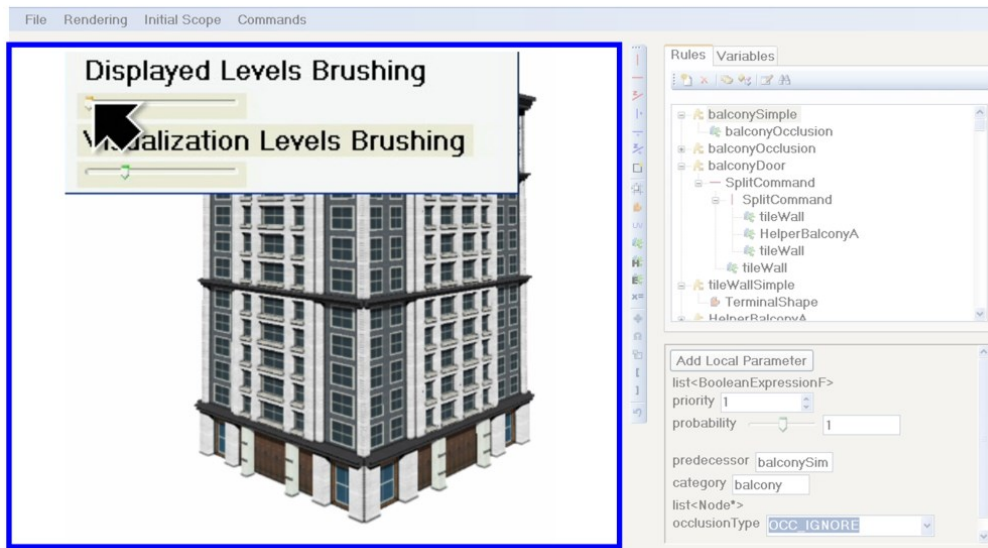
- Visual interface to define rules and their parameters
- Videos from Lipp2008

## Interactive rule visualization



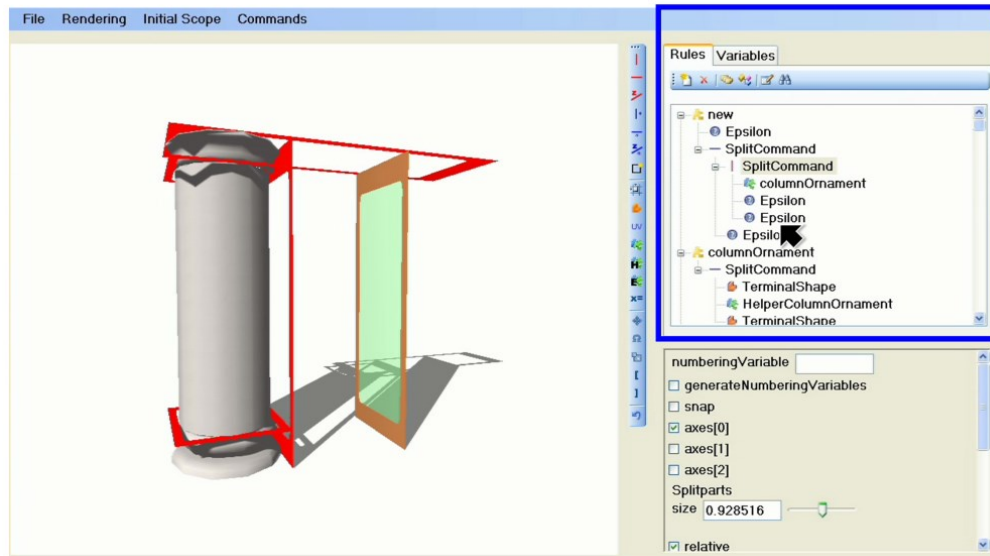
Video: Lipp 2008

## Focus & context editing



Video: Lipp 2008

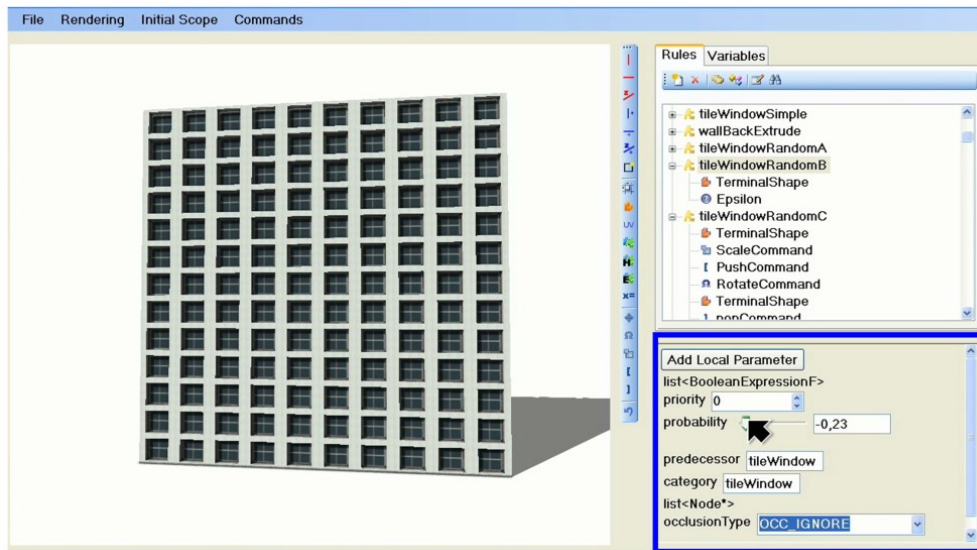
## Linked tree-view: rule explorer



Video: Lipp 2008



## Parameter view



Video: Lipp 2008

## Graph-based rule editors

- Examples: Silva2015, Thaller2012, Thaller2013, Patow2012, Houdini

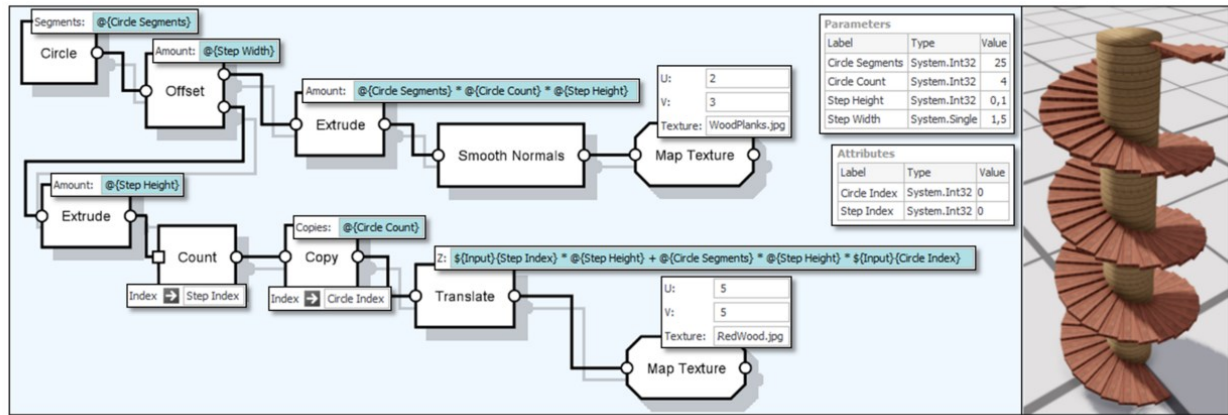


Image: Silva 2015

## Graph-based rule editors

- Naming rules is easier
- Specifying data flow has multiple challenges, e.g.
  - How to implement derivation control, e.g. construction stages?
  - How to implement recursion?
  - How to query context?

## Manipulators

- Extend **dimension lines** used in technical drawing
- A **manipulator** is a user interface element that enables the interactive manipulation of parameters
  - Length parameters
  - Angles
  - Discrete parameters
  - ...

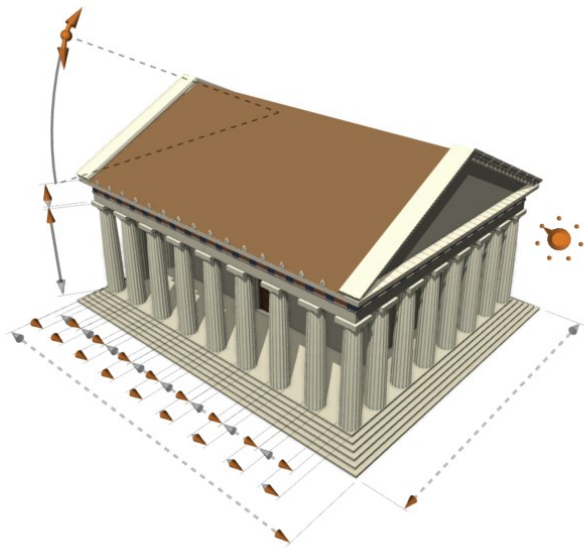


Image: Kelly 2015

## Manipulators and handles

- How to place manipulators while the view is changing?
  - Static in 3D
  - Dynamic [\[Kelly2015\]](#)
  - Specify view points [\[Krecklau2012\]](#)
- How to place handles and manipulators in real time?
  - Greedy / energy function [\[Kelly2015\]](#)
  - Global optimization

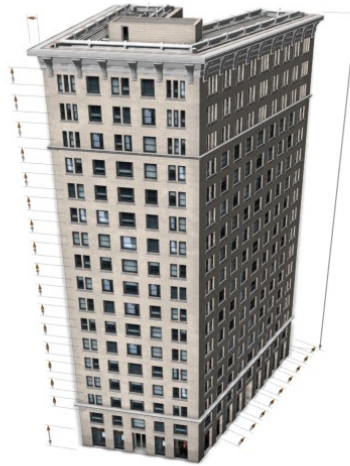


Image: Kelly 2015

## Manipulators and handles

- How to specify placement parameters?
  - Automatically [\[Open Problem\]](#)
  - Manually in the grammar [\[Kelly2015, Krecklau2012\]](#)
- How to decide what parameters should have manipulators?
  - Automatically [\[Open Problem\]](#)
  - Manually in the grammar [\[Kelly2015, Krecklau2012\]](#)



Image: Kelly 2015



Many commercial systems suffer from world-static dimension lines

Video: Kelly 2015

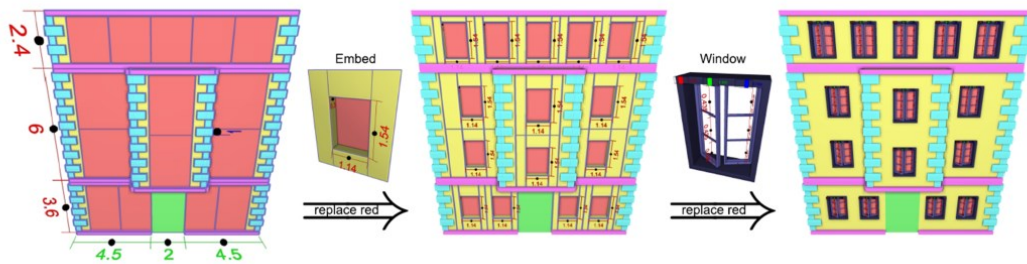
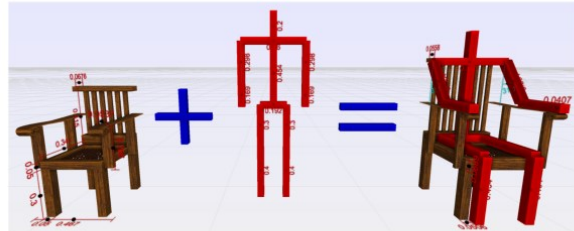
## Procedural high-level primitives

- Krecklau2012
- P-Mode: full grammar-based procedural modeling (e.g. scripting)
- **High-level primitives** (HL-primitives):
  - modules with a fixed set of parameters
  - manipulators
  - camera views
- Manipulators and Camera Views are specified by grammar extensions



## High-level primitives examples

- Human stick figure controls chair parameters
- Façade editing workflow



Images: Krecklau 2012

## Styles

- Concept used in CityEngine
- Multiple styles in a rules file
- Styles define attributes and rules
- Styles can be derived from other styles
  - All attributes and rules are inherited from the parent style per default

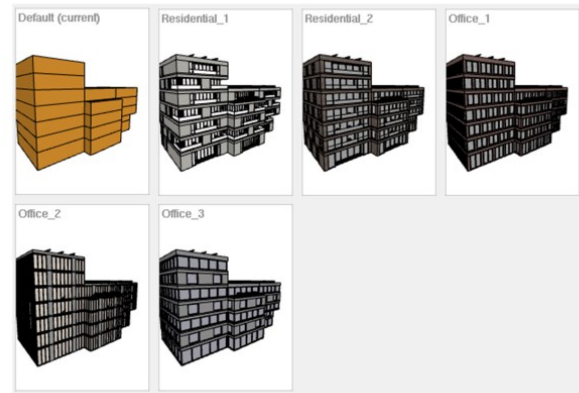


Image: CityEngine 2015 Online Help System

## Style example

```
attr height = 10
attr type = "residential"
Lot --> MassModel(height, type)

style Commercial
attr height = 5
attr type = "commercial"

style Commercial_Restaurant extends Commercial
attr height = 3.5
```

## Design galleries

- Seminal paper [\[Marks1997\]](#)
  - Sample the design space of a parametric model
  - Visualize the results
- How to compare two models of the design space?
  - Compare parameters of the models directly
  - Compare renderings of the models [\[Lienhard2014\]](#)
  - Compare extracted features of the models [\[Dang2015\]](#)
- How to present the results?
  - Hierarchical vs. non-hierarchical
  - Table vs. star vs. dimension reduction
- Most relevant research published outside of procedural modeling

## Thumbnail galleries

- Proposed Solution [Lienhard2014]

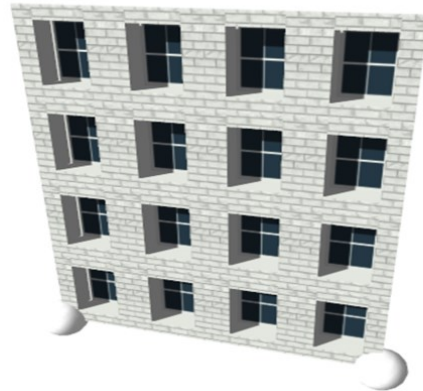
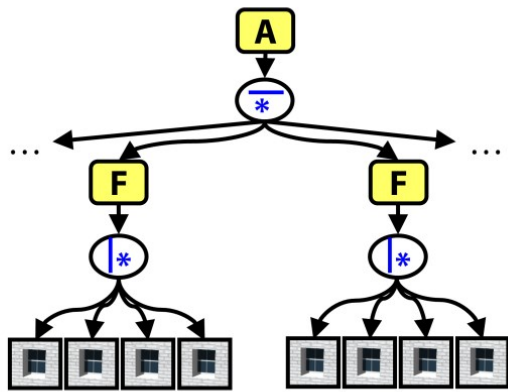


Image: Lienhard 2014

## Local edits: challenges

- **Persistence:**  
when locally editing a procedural model, how to preserve edits?
- **Selection:**  
how to make semantic, hierarchical selections?

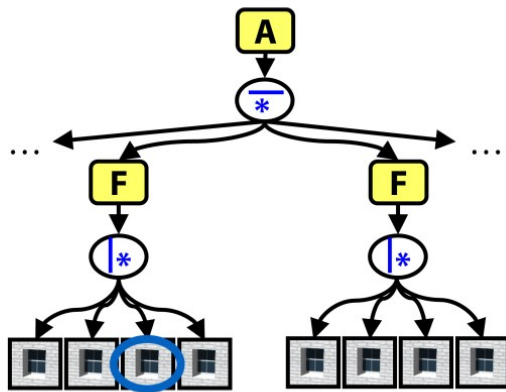
## Challenge: selection



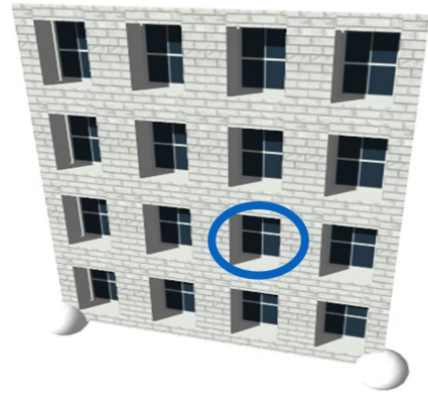
uniquely identify subset of shapes

Images: Lipp 2008

## Challenge: selection



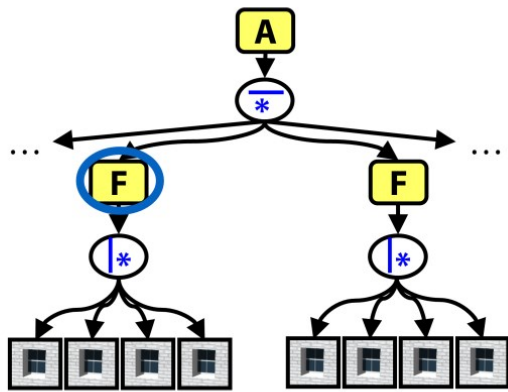
single shape



Images: Lipp 2008



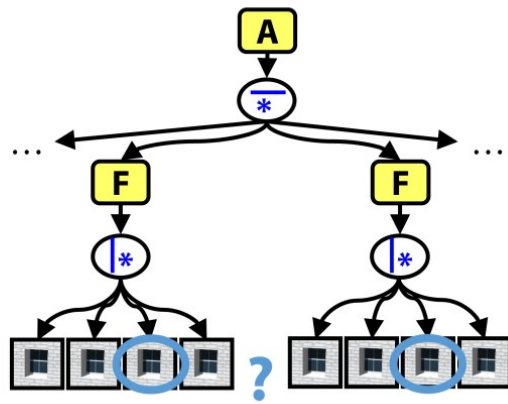
## Challenge: selection



hierarchical selection

Images: Lipp 2008

## Challenge: selection



no single shape represents selection  
semantic selection, property "column"



Images: Lipp 2008

## Challenge: persistence

- Workflow:

- User makes a **local edit E1**  
e.g. add a balcony



Video: Lipp 2008

## Challenge: persistence

- Workflow:
  - User makes a **local edit E1**  
e.g. add a balcony
  - User makes a second **local edit E2**  
e.g. change window parameter
- How to ensure edit **E1** is preserved?

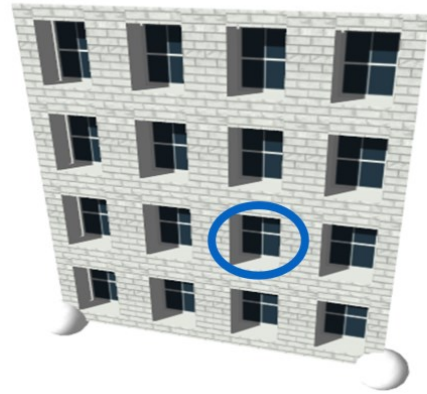
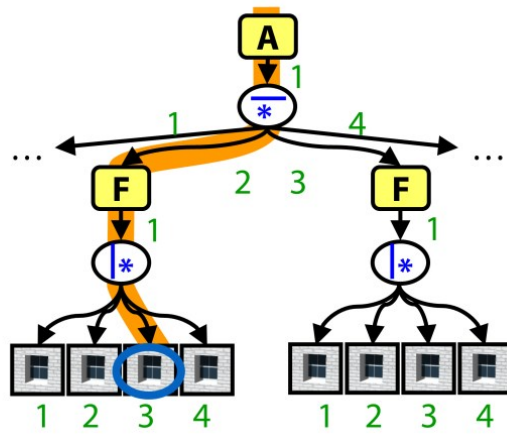


Video: Lipp 2008

## Local edits: solutions

- Semantic tags and instance locators
- Expressions within grammars
- Exception nodes

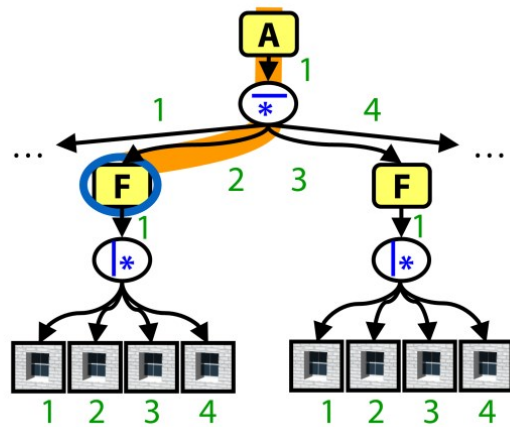
## Exact instance locator



exact locator = { **A**, 1, **\***, 2, **F**, 1, **\***, 3,  }

Image: Lipp 2008

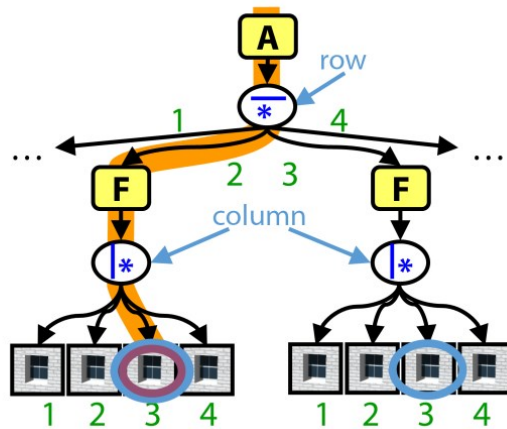
## Exact instance locator



exact locator = { **A**, 1, **\***, 2, **F** }

Image: Lipp 2008

## Semantic instance locator



- semantic locator = { row = 2, column = 3 }
- semantic locator = { row = \*, column = center+1 }

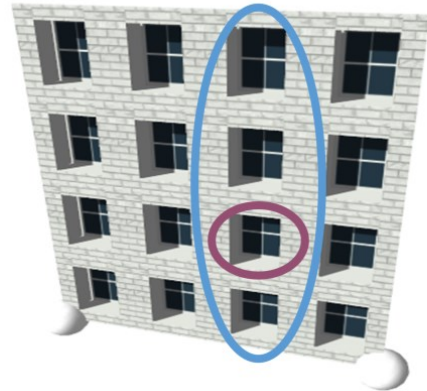


Image: Lipp 2008



## How to use instance locators?

- Use an external algorithm to modify a result after the derivation [Lipp2008]
- Query instance locators in a graph-based modeling system [Patow2012]
- Manually write conditional rules [CityEngine]
- Automatically change the rule base according to user input

## Expression within grammars

- Manually extend a grammar using instance locators
- E.g., CityEngine operation `getTreeKey`
- Use `getTreeKey` to write conditional rules
- `getTreeKey` returns a sequence of numbers
- Marked floor would have a key 0 – 0 - 1

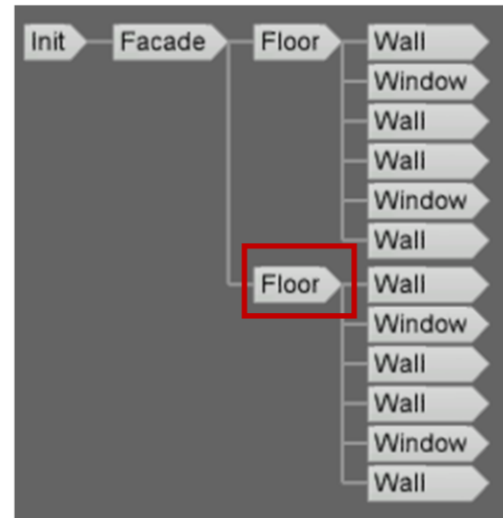


Image: CityEngine online help

## Exception nodes

- Filter data stream in graph based procedural modeling [Patow2012]
- e.g. filter based on instance locators

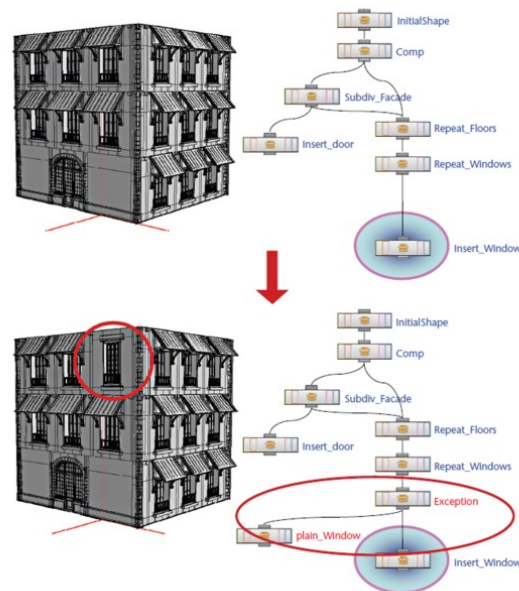


Image: Patow 2012

## Parameter adjustments via feedback loops

- Scripting and reporting
- Coupling with physical simulation
- User-based preference scores
- Optimization-based parameter tuning
- Optimization-based grammar derivation

## Scripting and reporting

- Write a grammar using a reporting function
  - E.g. `FloorArea --> report("area", geometry.area)`
- Analyze the output / the report
- Feedback
  - Change the rules of the grammar manually
  - Write a script, e.g. Python, to post-process the result
  - Write a script to modify the grammar

## Coupling with physical simulation

### ■ Components:

- Discrete sampling / optimization to suggest different variations
- Continuous optimization to modify model parameters



### ■ Example:

- Whiting2009
- Compute stable masonry structures
- Focuses on parameter search only

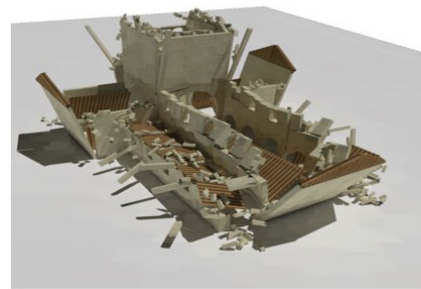
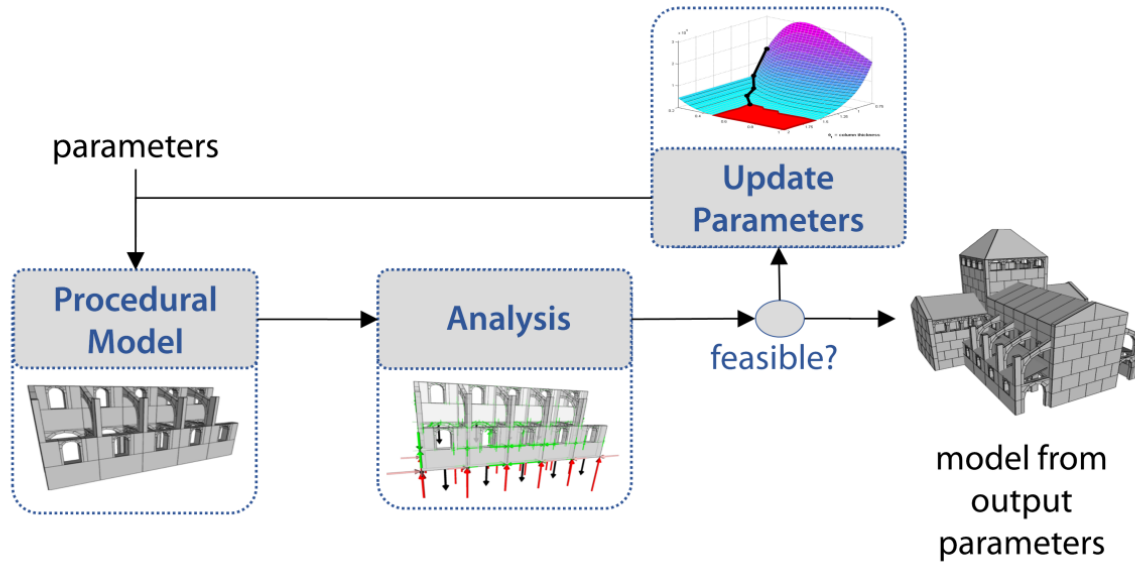


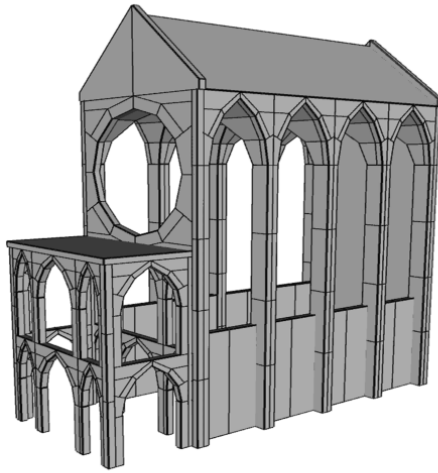
Image: Whiting 2009

## Optimization framework



Images: Whiting 2009

## Typical parameters



- building height
- thickness of columns, walls, arches
- window size
- angle of flying buttresses

Image: Whiting 2009



## User-based preference scores

- A stochastic grammar samples from a distribution  $P(M)$
- Goal: adjust the **parameters** and **structure** of the grammar so that the distribution matches user specified preference scores
- Proposed Solution [Dang2015]:
  - User scores generated models
  - Gaussian Process Regression to interpolate scores
  - Parameter and Structure Learning to adapt the grammar

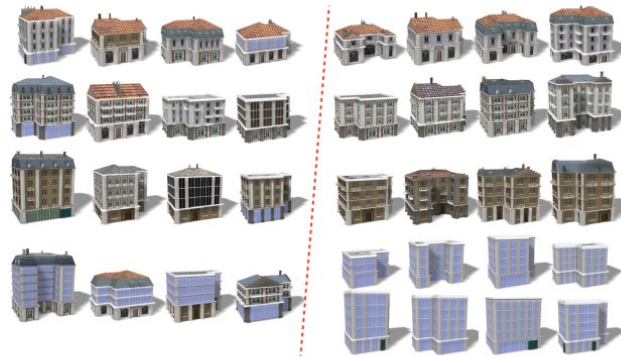
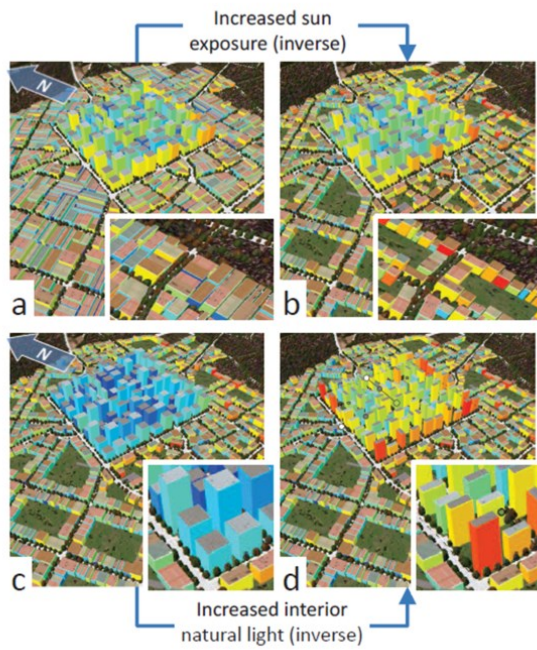
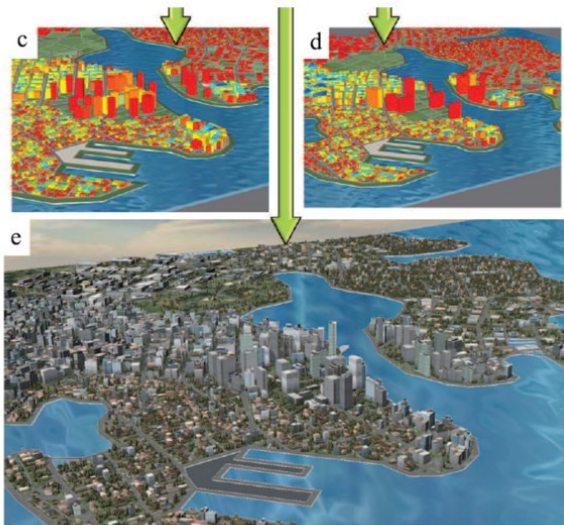


Image: Dang 2015

## Optimization-based parameter tuning

- Grammar has (geometric) parameters, e.g.,
  - Mean and std. deviation of height
  - Front and side setback from the road
  - Maximum front width
  - Maximum depth
- Indicator functions are higher level goal functions for the design, e.g.
  - Floor area ratio
  - Sunlight exposure
  - Visibility of landmarks
- How to tune grammar parameters to optimize goal functions?
- Proposed Solution [[Vanegas2012](#)]
  - MCMC
  - Neural Networks

## Design examples

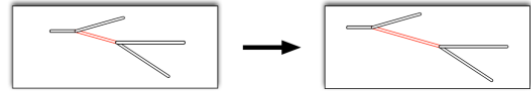


Images: Vanegas2012

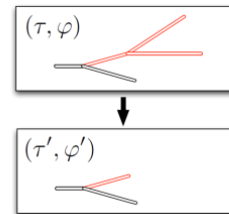
## Optimization-based grammar derivation

- Goal: Instead of sampling from the natural grammar distribution  $P(M)$ , how to sample from  $P(M)*F(M)$ 
  - $F(M)$  is some external function to optimize, e.g. fit inside a volume
- Proposed solution [Talton2011]
  - rjMCMC

- Diffusion move (change parameters)



- Jump move (change structure)



## Design example

- Trees should grow inside given envelopes

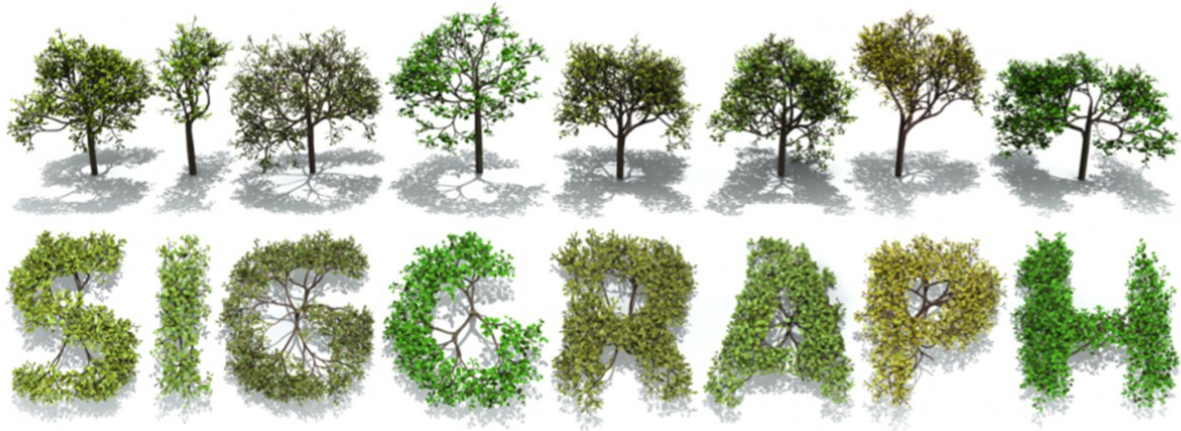


Image: Talton2011

## GPU-based variants

- Challenges
- Simplifications
- Fragment-wise grammar evaluation
- Instantiation of detailed asset geometry
- Generation of actual geometry to be rendered

## Challenges

- Context-sensitive rules are difficult to parallelize
- At the beginning of the derivation, not a lot of shapes are present
- Derivation is recursive
- Parallelizing on the GPU needs data that can be processed with the same instructions (SIMD)

## Simplifications

- Limit recursions
- Limit context-sensitive rules
- Limit the geometry of non-terminal shapes
- Limit operations to a subset

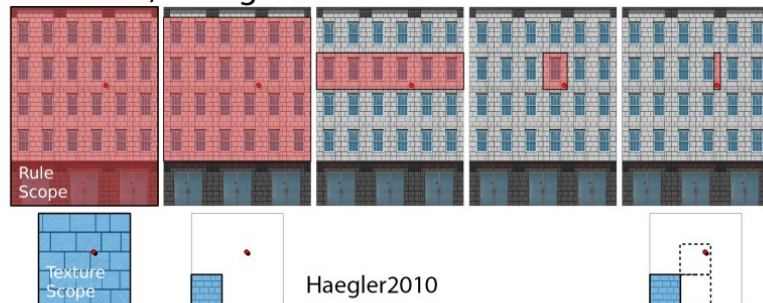


## Fragment-wise grammar evaluation

- Use ray-casting / ray-tracing and derive geometry along the ray
- Rendering is similar to ray tracing using hierarchical bounding volumes
- Requires a hierarchical grammar with some guarantees
- Examples: Haegler2010, Marvie2011, Kuang2013



Kuang2013



Haegler2010

## Instantiation of detailed asset geometry

- Fragment shader can request detailed assets
- Assets are transformed and scaled appropriately
- Several techniques ensure correct visibility

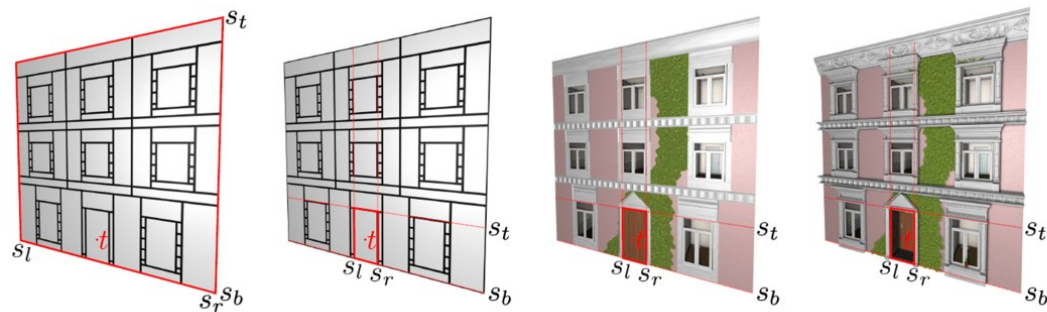


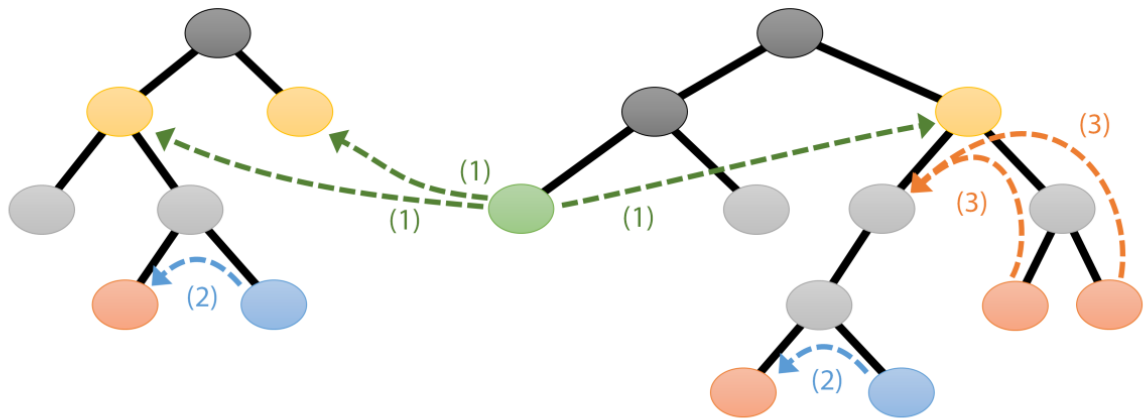
Image: Krecklau2013

## Generation of geometry on the GPU

- Steinberger et al. EG 2014 ([PGA](#))
- Support for context-sensitive queries on the GPU
- Rule scheduling to reduce kernel overhead
- Rule grouping to reduce divergence & global memory accesses
- Operator level parallelism to increase performance
- Alternative Solution: Marvie et al. 2012

## Context sensitivity in PGA

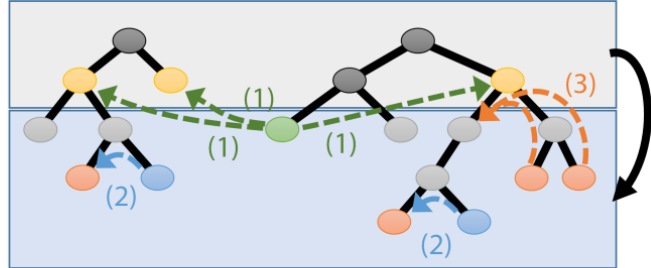
- Three mechanisms to express dependencies



## Context sensitivity in PGA

### 1. Evaluation Phases

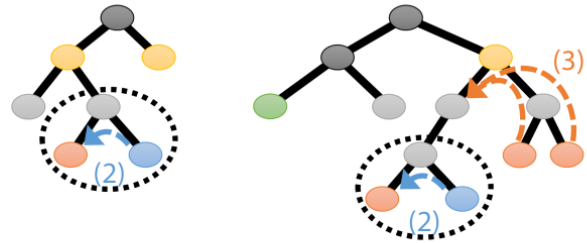
- Similar to CGA-priorities
  - No common predecessor
  - Large distance between nodes in the tree
  - e.g. is there a higher building
- 
- Requires complete synchronization between phases
  - Most costly way of context-sensitivity
  - One set of shapes for each phase
    - Only continue to next set if previous has been completed



## Context sensitivity in PGA

### 2. Sibling Queries

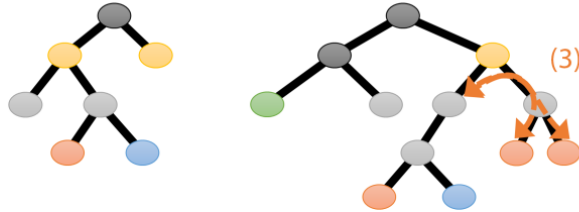
- Shapes involved share the same parent
- No need to synchronize globally
- e.g. is there a neighboring facade tile to both sides → it the tile at a corner
- Evaluate query in parent and pass to all children



## Context sensitivity in PGA

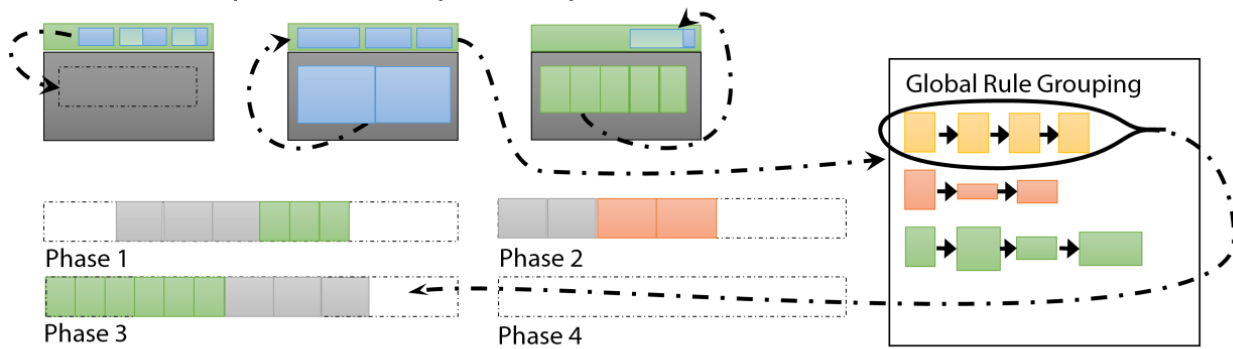
### 3. Bilateral Evaluation Queries

- Involved shapes have a common predecessor
  - Distance between the nodes is not too big
  - e.g. is there a balcony in front of the window → create a door instead
- 
- Preprocessing: find common predecessor
  - Derivation: pass the common predecessor along the path and duplicate the derivation process → independent of evaluation order



## Rule scheduling

- One queue per evaluation phase
- Local rule grouping
- Global rule grouping
- Workers pull from and push to queues





## Operator-level parallelism

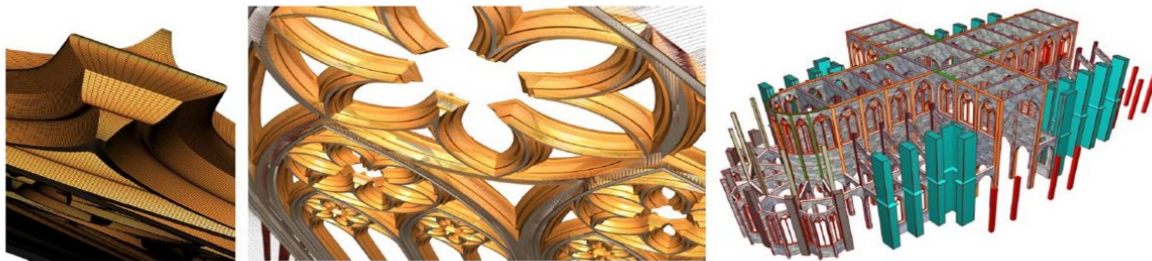
- Operators execute same operation multiple times
  - e.g. repeatX creates 20 identical boxes
  - Make use of parallelism within operators
  - → use 20 threads for repeat, etc.
  
- More parallelism → more performance
- Equal operations → less divergence
- Better memory access patterns → more performance
- Require fewer shapes to fill block → more efficient local queuing

## Background: other modeling approaches

- Component-based modeling
- Generative modeling:
  - GML
  - Bentley Generative Components
  - Boundary solid grammars

## Generative modeling language

- “Postscript for Meshes”
- Stack-based mesh modeling
- Operators take parameters from the stack
- Focus is different
  - GML: generation of detailed assets
  - CGA-shape: arrangement of assets

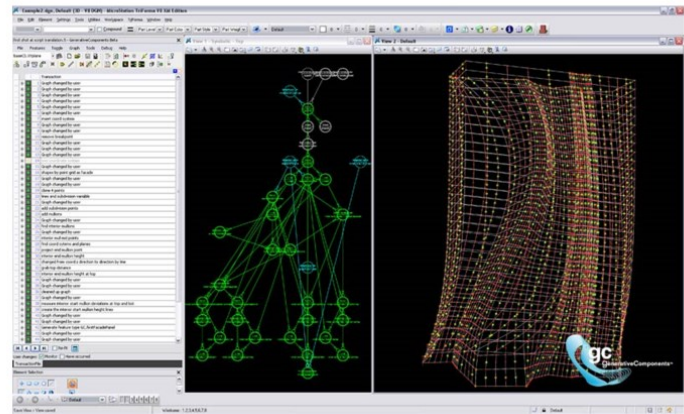


## Bentley Generative Components

- Graph-based editing
- Good for free form architecture



HOK and Bruno Hoppold

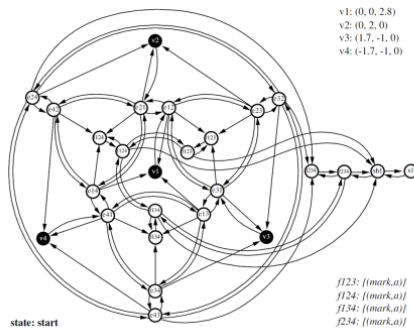


joshnelly.com

Bentley Generative Components is popular for modeling free-form architecture. For example, stadiums.

## Boundary solid grammars

- Heisserman
- Operates on b-graphs
  - vertex, edgeloop, face, shell, solid



- Rules
  - Primitive match conditions: matching in the b-graph
  - Primitive operations: modifying the b-graph
  - Logic rules / predicates: combine primitive match conditions or primitive operations



## 6 Conclusions

Course: Practical Grammar-based Procedural Modeling of Architecture

### Conclusions

Peter Wonka



## Conclusions

- Procedural modeling is currently the best available tool for large-scale urban modeling of virtual cities
- Many challenges remain in the basic technology and in advanced topics



## Example challenges

- Alignment of architectural elements
- Size independent modeling of medium and complex layouts
- Coordination of elements in different façade parts, e.g. windows in gabled roofs
- Inverse procedural modeling
- Using procedural modeling in computer vision
- Rule design without programming



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